

# Comparison of VCE Mathematics and the new IB Mathematics Courses

By Tracey Blunden, Beatrice Roberts, Evan Pearson and Bin Wang

As discussed in our brief we have compared the Victorian Curriculum to the New IB Mathematics Courses. Cross referencing the VCE standards with the four new IB Mathematics courses. There are a number of the IB Standards from the four courses that are not covered in the VCE. These have been added at the end.

### **VCE General/Further Mathematics**

Below is the table that cross references the VCE General and Further Mathematics. Please note for unit 1/2 it is called General Mathematics and Unit 3/4 is called Further Mathematics.

| Course               | Unit | Content                                                                                            | Included in Prior learning | Included in Analysis SL       | Included in Analysis HL       | Included in Apps SL           | Included in Apps HL           | Comments |
|----------------------|------|----------------------------------------------------------------------------------------------------|----------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|----------|
| VCE General Unit 1/2 | AOS1 | substitution into, and transposition of linear relations, such as scale conversion                 | √                          |                               |                               |                               |                               |          |
| VCE General Unit 1/2 | AOS1 | solution of linear equations, including literal linear equations                                   | √                          |                               |                               |                               |                               |          |
| VCE General Unit 1/2 | AOS1 | developing formulas from word descriptions and substitution of values into formulas and evaluation | √                          |                               |                               |                               |                               |          |
| VCE General Unit 1/2 | AOS1 | construction of tables of values from a given formula                                              | √                          |                               |                               |                               |                               |          |
| VCE General Unit 1/2 | AOS1 | linear relations defined recursively and simple applications of these relations                    |                            | √*<br>Recursion not mentioned | √*<br>Recursion not mentioned | √*<br>Recursion not mentioned | √*<br>Recursion not mentioned |          |

|                            |      |                                                                                                                                                                                                                                                                                                            |   |   |   |   |   |  |
|----------------------------|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|---|---|--|
| VCE<br>General<br>Unit 1/2 | AOS1 | numerical, graphical and algebraic solutions of simultaneous linear equations in two variables                                                                                                                                                                                                             | √ |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS1 | use of linear equations, including simultaneous linear equations in two variables, and their application to solve practical problems.                                                                                                                                                                      | √ |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS2 | review of computation: order of operations, directed numbers, scientific notation, estimation, exact and approximate answers, rounding correct to a given number of decimal places or significant figures                                                                                                  | √ |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS2 | efficient mental and by-hand estimation and computation in relevant contexts                                                                                                                                                                                                                               | √ |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS2 | orders of magnitude, units of measure that range over multiple orders of magnitude and their use and interpretation, and the use and interpretation of log to base 10 scales, such as the Richter scale                                                                                                    |   | √ | √ | √ | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS2 | use of ratios and proportions, and percentages and percentage change to solve practical problems                                                                                                                                                                                                           | √ |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS2 | the unitary method and its use to make comparisons and solve practical problems involving ratio and proportion.                                                                                                                                                                                            | √ |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS2 | percentage increase and decrease applied to various financial contexts such as the price to earnings ratios of shares and percentage dividends, determining the impact of inflation on costs and the spending power of money over time, calculating percentage mark-ups and discounts, and calculating GST | √ |   |   |   |   |  |

|                            |      |                                                                                                                                                                                                                                                                                |  |   |   |   |   |  |
|----------------------------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|--|
| VCE<br>General<br>Unit 1/2 | AOS2 | applications of simple interest and compound interest                                                                                                                                                                                                                          |  | √ | √ | √ | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS2 | cash flow in common savings and credit accounts including interest calculation                                                                                                                                                                                                 |  |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS2 | compound interest investments and loans                                                                                                                                                                                                                                        |  | √ | √ | √ | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS2 | comparison of purchase options including cash, credit and debit cards, personal loans, and time payments (hire purchase).                                                                                                                                                      |  |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | use of matrices to store and display information that can be presented in a rectangular array of rows and columns such as databases and links in social and road networks                                                                                                      |  |   |   |   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | types of matrices (row, column, square, zero and identity) and the order of a matrix                                                                                                                                                                                           |  |   |   |   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | matrix addition, subtraction, multiplication by a scalar, and matrix multiplication including determining the power of a square matrix using technology as applicable                                                                                                          |  |   |   |   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | use of matrices, including matrix products and powers of matrices, to model and solve problems, for example costing or pricing problems, and squaring a matrix to determine the number of ways pairs of people in a network can communicate with each other via a third person |  |   |   |   | √ |  |

|                            |      |                                                                                                                                                                                                                  |  |   |   |   |   |  |
|----------------------------|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|--|
| VCE<br>General<br>Unit 1/2 | AOS3 | inverse matrices and their applications including solving a system of simultaneous linear equations.                                                                                                             |  |   |   |   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | introduction to the notations, conventions and representations of types and properties of graphs, including edge, loop, vertex, the degree of a vertex, isomorphic and connected graphs and the adjacency matrix |  |   |   |   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | description of graphs in terms of faces (regions), vertices and edges and the application of Euler's formula for planar graphs                                                                                   |  |   |   |   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | connected graphs: walks, trails, paths, cycles and circuits with practical applications                                                                                                                          |  |   |   |   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | weighted graphs and networks, and an introduction to the shortest path problem (solution by inspection only) and its practical application                                                                       |  |   |   |   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | trees and minimum spanning trees, Prim's algorithm, and their use to solve practical problems.                                                                                                                   |  |   |   |   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | the concept of a sequence as a function                                                                                                                                                                          |  | √ | √ | √ | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | use of a first-order linear recurrence relation to generate the terms of a number sequence                                                                                                                       |  |   |   |   |   |  |

|                            |      |                                                                                                                                                                                                                                                                                                                                      |  |                            |                            |                            |                            |  |
|----------------------------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|----------------------------|----------------------------|----------------------------|----------------------------|--|
| VCE<br>General<br>Unit 1/2 | AOS3 | tabular and graphical display of sequences                                                                                                                                                                                                                                                                                           |  | √                          | √                          | √                          | √                          |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | generation of an arithmetic sequence using a recurrence relation, tabular and graphical display; and the rule for the nth term of an arithmetic sequence and its evaluation                                                                                                                                                          |  | √*recurrence not mentioned | √*recurrence not mentioned | √*recurrence not mentioned | √*recurrence not mentioned |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | use of a recurrence relation to model and analyse practical situations involving discrete linear growth or decay such as a simple interest loan or investment, the depreciating value of an asset using the unit cost method; and the rule for the value of a quantity after n periods of linear growth or decay and its use.        |  | √*recurrence not mentioned | √*recurrence not mentioned | √*recurrence not mentioned | √*recurrence not mentioned |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | generation of a geometric sequence using a recurrence relation and its tabular or graphical display; and the rule for the nth term and its evaluation                                                                                                                                                                                |  | √*recurrence not mentioned | √*recurrence not mentioned | √*recurrence not mentioned | √*recurrence not mentioned |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | use of a recurrence relation to model and analyse practical situations involving geometric growth or decay such as the growth of a compound interest loan, the reducing height of a bouncing ball, reducing balance depreciation; and the rule for the value of a quantity after n periods of geometric growth or decay and its use. |  | √*recurrence not mentioned | √*recurrence not mentioned | √*recurrence not mentioned | √*recurrence not mentioned |  |
| VCE<br>General<br>Unit 1/2 | AOS3 | generation of the Fibonacci and similar sequences using a recurrence relation, tabular and graphical display                                                                                                                                                                                                                         |  | √*TOK Links                | √*TOK Links                | √*TOK Links                | √*TOK Links                |  |

|                            |      |                                                                                                                                                                         |                             |                |                |                |                |  |
|----------------------------|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------|----------------|----------------|----------------|----------------|--|
| VCE<br>General<br>Unit 1/2 | AOS3 | use of Fibonacci and similar sequences to model and analyse practical situations                                                                                        |                             | √*TOK<br>Links | √*TOK<br>Links | √*TOK<br>Links | √*TOK<br>Links |  |
| VCE<br>General<br>Unit 1/2 | AOS4 | review of units of measurement of length, angle, area, volume and capacity                                                                                              | √                           |                |                |                |                |  |
| VCE<br>General<br>Unit 1/2 | AOS4 | Pythagoras' theorem in two dimensions, and simple examples in three dimensions, and application to practical problems                                                   | √                           |                |                |                |                |  |
| VCE<br>General<br>Unit 1/2 | AOS4 | perimeter and areas of triangles (including the use of Heron's formula), quadrilaterals, circles and composite shapes and practical applications                        | * Not<br>Heron's<br>formula |                |                |                |                |  |
| VCE<br>General<br>Unit 1/2 | AOS4 | volumes and surface areas of solids (spheres, cylinders, pyramids and prisms and their composites) and practical applications                                           | √                           |                |                |                |                |  |
| VCE<br>General<br>Unit 1/2 | AOS4 | similar figures including the mathematical conditions for similarity of two-dimensional shapes, and the linear scale factor and its extension to areas and volumes      |                             |                |                |                |                |  |
| VCE<br>General<br>Unit 1/2 | AOS4 | similarity of solids and the application of linear scale factor $k > 0$ to scale lengths, surface areas and volumes with practical applications.                        |                             |                |                |                |                |  |
| VCE<br>General<br>Unit 1/2 | AOS4 | review of the use of trigonometric ratios for sine, cosine and tangent to find the length of an unknown side or the size of an unknown angle in a right-angled triangle | √                           |                |                |                |                |  |

|                            |      |                                                                                                                                                                                                                    |   |   |   |                     |   |  |
|----------------------------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|---------------------|---|--|
| VCE<br>General<br>Unit 1/2 | AOS4 | application of the trigonometry of right-angled triangles to solve practical problems including the use of angles of elevation and depression, and the use of three figure (true) bearings in navigation           | √ |   |   |                     |   |  |
| VCE<br>General<br>Unit 1/2 | AOS4 | extension of sine and cosine to angles of up to $180^\circ$                                                                                                                                                        |   | √ | √ | √                   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS4 | area of a triangle using the rule $\text{Area} = \frac{1}{2} ab \sin C$                                                                                                                                            |   | √ | √ | √                   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS4 | the sine rule (including the ambiguous case) and cosine rule (as a generalisation of Pythagoras' theorem) and their application to solving practical problems requiring the solution of non-right angled triangles |   | √ | √ | √*No ambiguous case | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS4 | sets of sufficient information to determine a triangle.                                                                                                                                                            |   | √ | √ | √                   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | review of linear functions and graphs                                                                                                                                                                              |   | √ | √ | √                   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | the concept of a linear model and its specification                                                                                                                                                                |   | √ | √ | √                   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | the construction of a linear model to represent a practical situation including domain of application                                                                                                              |   | √ | √ | √                   | √ |  |



|                            |      |                                                                                                                                                 |   |   |   |   |   |  |
|----------------------------|------|-------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|---|---|--|
| VCE<br>General<br>Unit 1/2 | AOS5 | the interpretation of the parameters of a linear model and its use to make predictions, including the issues of interpolation and extrapolation |   | √ | √ | √ | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | fitting a linear model to data by using the equation of a line fitted by eye                                                                    |   | √ | √ | √ | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | use of piecewise linear (line segment) graphs to model and analyse practical situations.                                                        |   |   |   |   | √ |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | linear inequalities in one and two variables and their graphical representation                                                                 | √ |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | the linear programming problem and its purpose                                                                                                  |   |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | the concepts of feasible region, constraint and objective function in the context of solving a linear programming problem                       |   |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | use of the corner-point principle to determine the optimal solution/s of a linear programming problem                                           |   |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | formulation and graphical solution of linear programming problems involving two variables.                                                      |   |   |   |   |   |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | numerical, graphical and algebraic approaches to direct, inverse and joint variation                                                            |   |   |   |   |   |  |

|                            |      |                                                                                                                                                                                                                                                                                           |   |                     |                     |                     |                     |  |
|----------------------------|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---------------------|---------------------|---------------------|---------------------|--|
| VCE<br>General<br>Unit 1/2 | AOS5 | transformation of data to linearity to establish relationships between variables, for example $y$ and $x^2$ , or $y$ and $\frac{1}{x}$                                                                                                                                                    |   |                     |                     |                     |                     |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | modelling of given non-linear data using the relationships $y = kx^2 + c$ and $y = kx + c$ where $k > 0$                                                                                                                                                                                  |   |                     |                     |                     |                     |  |
| VCE<br>General<br>Unit 1/2 | AOS5 | modelling of data using the logarithmic function $y = a \log_{10}(x) + c$ where $a > 0$ .                                                                                                                                                                                                 |   |                     |                     |                     | ✓                   |  |
| VCE<br>General<br>Unit 1/2 | AOS6 | types of data, including categorical (nominal or ordinal) or numerical (discrete or continuous)                                                                                                                                                                                           |   | ✓                   | ✓                   | ✓                   | ✓                   |  |
| VCE<br>General<br>Unit 1/2 | AOS6 | display and description of categorical data distributions using frequency tables and bar charts; and the mode and its interpretation                                                                                                                                                      | ✓ | ✓                   | ✓                   | ✓                   | ✓                   |  |
| VCE<br>General<br>Unit 1/2 | AOS6 | display and description of numerical data distributions in terms of shape, centre and spread using histograms, stem plots (including back-to-back stem plots) and dot plots and choosing between plots                                                                                    |   | ✓* No stem and leaf | ✓* No stem and leaf | ✓* No stem and leaf | ✓* No stem and leaf |  |
| VCE<br>General<br>Unit 1/2 | AOS6 | measures of centre and spread and their use in summarising numerical data distributions, including use of and calculation of the sample summary statistics, median, mean, range, interquartile range (IQR) and standard deviation; and choosing between the measures of centre and spread | ✓ |                     |                     |                     |                     |  |

|                            |      |                                                                                                                                                                                                                                                                                                   |  |                     |                     |                     |                     |  |
|----------------------------|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------------------|---------------------|---------------------|---------------------|--|
| VCE<br>General<br>Unit 1/2 | AOS6 | the five-number summary and the boxplot as its graphical representation and display, including the use of the lower fence ( $Q1 - 1.5 \times IQR$ ) and upper fence ( $Q3 + 1.5 \times IQR$ ) to identify possible outliers                                                                       |  | ✓                   | ✓                   | ✓                   | ✓                   |  |
| VCE<br>General<br>Unit 1/2 | AOS6 | use of back-to-back stem plots or parallel boxplots, as appropriate, to compare the distributions of a single numerical variable across two or more groups in terms of centre (median) and spread (IQR and range), and the interpretation of any differences observed in the context of the data. |  | ✓* No stem and leaf | ✓* No stem and leaf | ✓* No stem and leaf | ✓* No stem and leaf |  |
| VCE<br>General<br>Unit 1/2 | AOS6 | response and explanatory variables                                                                                                                                                                                                                                                                |  |                     |                     |                     |                     |  |
| VCE<br>General<br>Unit 1/2 | AOS6 | scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction, form and strength                                                                                                                                   |  | ✓                   | ✓                   | ✓                   | ✓                   |  |
| VCE<br>General<br>Unit 1/2 | AOS6 | the Pearson correlation coefficient $r$ , calculation and interpretation, and correlation and causation                                                                                                                                                                                           |  | ✓                   | ✓                   | ✓                   | ✓                   |  |
| VCE<br>General<br>Unit 1/2 | AOS6 | use of the least squares line to model an observed linear association and the interpretation of its intercept and slope in the context of the data                                                                                                                                                |  | ✓                   | ✓                   | ✓                   | ✓                   |  |
| VCE<br>General<br>Unit 1/2 | AOS6 | use of the model to make predictions and identify limitations of extrapolation.                                                                                                                                                                                                                   |  | ✓                   | ✓                   | ✓                   | ✓                   |  |

|                          |               |                                                                                                                                                                                                                                                                                                                                                                    |   |                                  |                                  |                                  |                                  |  |
|--------------------------|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----------------------------------|----------------------------------|----------------------------------|----------------------------------|--|
| VCE<br>Further<br>Unit 3 | Data Analysis | review of types of data                                                                                                                                                                                                                                                                                                                                            |   | ✓                                | ✓                                | ✓                                | ✓                                |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | review of representation, display and description of the distributions of categorical variables: data tables, two-way frequency tables and their associated segmented bar charts                                                                                                                                                                                   | ✓ | ✓                                | ✓                                | ✓                                | ✓                                |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | use of the distribution/s of one or more categorical variables to answer statistical questions                                                                                                                                                                                                                                                                     |   |                                  |                                  |                                  |                                  |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | review of representation, display and description of the distributions of numerical variables: dot plots, stem plots, histograms; the use of a log (base 10) scale to display data ranging over several orders of magnitude and their interpretation in powers of ten                                                                                              |   | ✓*No stem and leaf or log scales | ✓*No stem and leaf or log scales | ✓*No stem and leaf or log scales | ✓*No stem and leaf or log scales |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | summary of the distributions of numerical variables; the five-number summary and boxplots (including the use of the lower fence ( $Q1 - 1.5 \times IQR$ ) and upper fence ( $Q3 + 1.5 \times IQR$ ) to identify and display possible outliers); the sample mean and standard deviation and their use in comparing data distributions in terms of centre and spread |   | ✓                                | ✓                                | ✓                                | ✓                                |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | use of the distribution/s of one or more numerical variables to answer statistical questions                                                                                                                                                                                                                                                                       |   | ✓                                | ✓                                | ✓                                | ✓                                |  |

|                          |               |                                                                                                                                                                                                                                                                         |  |                     |                     |                     |                     |  |
|--------------------------|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------------------|---------------------|---------------------|---------------------|--|
| VCE<br>Further<br>Unit 3 | Data Analysis | the normal model for bell-shaped distributions and the use of the 68–95–99.7% rule to estimate percentages<br>and to give meaning to the standard deviation;<br>standardised values (z-scores) and their use in comparing data values across distributions              |  | ✓                   | ✓                   | ✓                   | ✓                   |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | population and sample, random numbers and their use to draw simple random samples from a population or<br>randomly allocate subjects to groups, the difference between population parameters (e.g., $\mu$ and $\sigma$ ), sample statistics (e.g., $\bar{x}$ and $s$ ). |  | ✓                   | ✓                   | ✓                   | ✓                   |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | response and explanatory variables and their role in investigating associations between variables                                                                                                                                                                       |  | ✓                   | ✓                   | ✓                   | ✓                   |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | contingency (two-way) frequency tables, two-way frequency tables and their associated bar charts (including<br>percentaged segmented bar charts) and their use in identifying and describing associations between two categorical variables                             |  |                     |                     |                     |                     |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | back-to-back stem plots, parallel dot plots and boxplots and their use in identifying and describing associations between a numerical and a categorical variable                                                                                                        |  | ✓* No stem and leaf | ✓* No stem and leaf | ✓* No stem and leaf | ✓* No stem and leaf |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction (positive/negative),                                                                                                       |  | ✓                   | ✓                   | ✓                   | ✓                   |  |

|                    |               |                                                                                                                                                                                                                                                                                 |  |   |   |   |   |  |
|--------------------|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|--|
|                    |               | form (linear/non-linear) and strength (strong/moderate/weak)                                                                                                                                                                                                                    |  |   |   |   |   |  |
| VCE Further Unit 3 | Data Analysis | answering statistical questions that require a knowledge of the associations between pairs of variables                                                                                                                                                                         |  |   |   | ✓ | ✓ |  |
| VCE Further Unit 3 | Data Analysis | Pearson correlation coefficient, $r$ , its calculation and interpretation                                                                                                                                                                                                       |  | ✓ | ✓ | ✓ | ✓ |  |
| VCE Further Unit 3 | Data Analysis | cause and effect; the difference between observation and experimentation when collecting data and the need for experimentation to definitively determine cause and effect                                                                                                       |  |   |   | ✓ | ✓ |  |
| VCE Further Unit 3 | Data Analysis | non-causal explanations for an observed association including common response, confounding, and coincidence; discussion and communication of these explanations in a particular situation in a systematic and concise manner.                                                   |  |   |   |   |   |  |
| VCE Further Unit 3 | Data Analysis | least squares line of best fit $y = a + bx$ , where $x$ represents the explanatory variable and $y$ represents the response variable; the determination of the coefficients $a$ and $b$ using technology, and the formulas $b = r \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$ |  | ✓ | ✓ | ✓ | ✓ |  |

|                          |               |                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |   |   |   |       |  |
|--------------------------|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|-------|--|
| VCE<br>Further<br>Unit 3 | Data Analysis | modelling linear association between two numerical variables, including the:<br>– identification of the explanatory and response variables<br>– use of the least squares method to fit a linear model to the data                                                                                                                                                                                                                    |  | √ | √ | √ | √     |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | interpretation of the slope and intercepts of the least squares line in the context of the situation being modelled,<br>including:<br>– use of the rule of the fitted line to make predictions being aware of the limitations of extrapolation<br>– use of the coefficient of determination, $r^2$ , to assess the strength of the association in terms of explained variation<br>– use of residual analysis to check quality of fit |  | √ | √ |   | √     |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | data transformation and its use in transforming some forms of non-linear data to linearity using a square, log or reciprocal transformation (on one axis only                                                                                                                                                                                                                                                                        |  |   |   |   | √*log |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | interpretation and use of the equation of the least squares line fitted to the transformed data to make predictions.                                                                                                                                                                                                                                                                                                                 |  |   |   |   | √*log |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | qualitative features of time series plots; recognition of features such as trend (long-term direction), seasonality (systematic, calendar related movements) and irregular fluctuations (unsystematic, short-term fluctuations);<br>possible outliers and their sources, including one-off real world events, and signs of structural change such                                                                                    |  |   |   |   |       |  |

|                          |               |                                                                                                                                                                                                                                                                                                      |  |  |  |  |  |  |
|--------------------------|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|--|--|--|
|                          |               | as<br>a discontinuity in the time series                                                                                                                                                                                                                                                             |  |  |  |  |  |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | numerical smoothing of time series data using moving means with consideration of the number of terms required (using centring when appropriate) to help identify trends in time series plot with large fluctuations                                                                                  |  |  |  |  |  |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | graphical smoothing of time series plots using moving medians (involving an odd number of points only) to help identify long-term trends in time series with large fluctuations                                                                                                                      |  |  |  |  |  |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | seasonal adjustment including the use and interpretation of seasonal indices and their calculation using seasonal and yearly means                                                                                                                                                                   |  |  |  |  |  |  |
| VCE<br>Further<br>Unit 3 | Data Analysis | modelling trend by fitting a least squares line to a time series with time as the explanatory variable (data de-seasonalised where necessary), and the use of the model to make forecasts (with re-seasonalisation where necessary) including consideration of the possible limitations of fitting a |  |  |  |  |  |  |



|                          |                          |                                                                                                                                                                                                                                                                                                                                                    |  |   |   |   |   |  |
|--------------------------|--------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|--|
|                          |                          | linear model and the limitations of extending into the future.                                                                                                                                                                                                                                                                                     |  |   |   |   |   |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | review of the use of a first-order linear recurrence relation to generate the terms of a sequence                                                                                                                                                                                                                                                  |  |   |   |   |   |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of a recurrence relation to model and compare (numerically and graphically) flat rate, unit cost and reducing balance depreciation of the value of an asset with time, including the use of a recurrence relation to determine the depreciating value of an asset after n depreciation periods, including from first principles for $n \leq 5$ |  |   |   |   |   |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of the rules for the future value of an asset after n depreciation periods for flat rate, unit cost and reducing balance depreciation and their application.                                                                                                                                                                                   |  | √ | √ | √ | √ |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | review of the concepts of simple and compound interest                                                                                                                                                                                                                                                                                             |  | √ | √ | √ | √ |  |

|                          |                          |                                                                                                                                                                                                                                                                                                                     |  |   |   |   |   |  |
|--------------------------|--------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|--|
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of a recurrence relation to model and analyse (numerically and graphically) a compound interest investment or loan, including the use of a recurrence relation to determine the value of the compound interest loan or investment after $n$ compounding periods, including from first principles for $n \leq 5$ |  |   |   |   |   |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | difference between nominal and effective interest rates and the use of effective interest rates to compare investment returns and the cost of loans when interest is paid or charged, for example, daily, monthly, quarterly                                                                                        |  |   |   |   |   |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | rule for the future value of a compound interest investment or loan after $n$ compounding periods and its use to solve practical problems                                                                                                                                                                           |  | ✓ | ✓ | ✓ | ✓ |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of a first-order linear recurrence relation to model and analyse (numerically and graphically) the amortisation of a reducing balance loan, including the use of a recurrence relation to determine the value of the loan or investment after $n$ payments, including from first principles for $n \leq 5$      |  |   |   |   |   |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of a table to investigate and analyse the amortisation of a reducing balance loan on a step-by-step basis, the payment made, the amount of interest paid, the reduction in the principal and the balance of the loan                                                                                            |  |   |   |   |   |  |

|                          |                          |                                                                                                                                                                                                                                                                                                                    |  |  |  |   |   |  |
|--------------------------|--------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|---|---|--|
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of technology with financial modelling functionality to solve problems involving reducing balance loans,<br>such as repaying a personal loan or a mortgage, including the impact of a change in interest rate on repayment amount, time to repay the loan, total interest paid and the total cost of the loan. |  |  |  |   |   |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of a first-order linear recurrence relation to model and analyse (numerically and graphically) the amortisation of an annuity, including the use of a recurrence relation to determine the value of the annuity after n payments,<br>including from first principles for $n \leq 5$                            |  |  |  |   |   |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of a table to investigate and analyse the amortisation of an annuity on a step-by-step basis, the payment made, the interest earned, the reduction in the principal and the balance of the annuity                                                                                                             |  |  |  | √ | √ |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of technology to solve problems involving annuities including determining the amount to be invested in an annuity to provide a regular income paid, for example, monthly, quarterly                                                                                                                            |  |  |  | √ | √ |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | simple perpetuity as a special case of an annuity that lasts indefinitely.                                                                                                                                                                                                                                         |  |  |  |   |   |  |

|                          |                          |                                                                                                                                                                                                                                                                                                                                                                                                 |  |  |  |  |   |  |
|--------------------------|--------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|--|---|--|
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of a first-order linear recurrence relation to model and analyse (numerically and graphically) annuity investment, including the use of a recurrence relation to determine the value of the investment after n payments have been made, including from first principles for $n \leq 5$                                                                                                      |  |  |  |  |   |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of a table to investigate and analyse the growth of an annuity investment on a step-by-step basis after each payment is made, the payment made, the interest earned and the balance of the investment                                                                                                                                                                                       |  |  |  |  |   |  |
| VCE<br>Further<br>Unit 3 | Finance and<br>Recursion | use of technology with financial modelling functionality to solve problems involving annuity investments, including determining the future value of an investment after a number of compounding periods, the number of compounding periods for the investment to exceed a given value and the interest rate or payment amount needed for an investment to exceed a given value in a given time. |  |  |  |  |   |  |
| VCE<br>Further<br>Unit 4 | Matrices                 | review of matrix arithmetic: the order of a matrix, types of matrices (row, column, square, diagonal, symmetric, triangular, zero, binary and identity), the transpose of a matrix, elementary matrix operations (sum, difference, multiplication of a scalar, product and power)                                                                                                               |  |  |  |  | v |  |

|                          |          |                                                                                                                                                                                                                                                                                                                                                                                                                                           |  |  |  |  |   |  |
|--------------------------|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|--|---|--|
| VCE<br>Further<br>Unit 4 | Matrices | inverse of a matrix, its determinant, and the condition for a matrix to have an inverse                                                                                                                                                                                                                                                                                                                                                   |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Matrices | use of matrices to represent numerical information presented in tabular form, and the use of a rule for the $a_{ij}$ th element of a matrix to construct the matrix                                                                                                                                                                                                                                                                       |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Matrices | binary and permutation matrices, and their properties and applications                                                                                                                                                                                                                                                                                                                                                                    |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Matrices | communication and dominance matrices and their use in analysing communication systems and ranking players in round-robin tournaments                                                                                                                                                                                                                                                                                                      |  |  |  |  |   |  |
| VCE<br>Further<br>Unit 4 | Matrices | use of matrices to represent systems of linear equations and the solution of these equations as an application of the inverse matrix; the concepts of dependent systems of equations and inconsistent systems of equations in the context of solving pairs of simultaneous equations in two variables; the formulation of practical problems in terms of a system of linear equations and their solution using the matrix inverse method. |  |  |  |  | √ |  |

|                          |                          |                                                                                                                                                                                                                                                                                                                                                                                                                         |  |  |  |  |   |  |
|--------------------------|--------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|--|---|--|
| VCE<br>Further<br>Unit 4 | Matrices                 | use of the matrix recurrence relation: $S_0$<br>= initial state matrix, $S_{n+1} = TS_n$ where T is a<br>transition matrix and $S_n$<br>is<br>a column state matrix, to generate a sequence of<br>state matrices, including in the case of regular<br>transition<br>matrices an informal identification of the equilibrium<br>state matrix (recognised by no noticeable change<br>from one<br>state matrix to the next) |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Matrices                 | use of transition diagrams, their associated transition<br>matrices and state matrices to model the transitions<br>between states in discrete dynamical situations and<br>their application to model and analyse practical<br>situations<br>such as the modelling and analysis of an insect<br>population comprising eggs, juveniles and adults                                                                         |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Matrices                 | use of the matrix recurrence relation $S_0$<br>= initial state matrix, $S_{n+1} = TS_n$<br>+ B to extend the modelling to populations<br>that include culling and restocking.                                                                                                                                                                                                                                           |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | a review of the concepts, conventions and<br>terminology of graphs including planar graphs and<br>Euler's rule, and<br>directed (digraphs) and networks                                                                                                                                                                                                                                                                 |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | use of matrices to represent graphs, digraphs and<br>networks and their application.                                                                                                                                                                                                                                                                                                                                    |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | review of the concepts, conventions and notations of<br>walks, trails, paths, cycles and circuits                                                                                                                                                                                                                                                                                                                       |  |  |  |  | √ |  |

|                          |                          |                                                                                                                                                                 |  |  |  |  |   |  |
|--------------------------|--------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|--|---|--|
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | eulerian trails and eulerian circuits: the conditions for a graph to have an eulerian trail or an eulerian circuit, properties and applications                 |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | hamiltonian paths and cycles: properties and applications.                                                                                                      |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | review of the basic concepts of trees and spanning trees                                                                                                        |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | minimum spanning trees in a weighted connected graph and their determination either by inspection or by using Prim's algorithm for larger scale problems        |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | use of minimal spanning trees to solve minimal connector problems.                                                                                              |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | use of networks to model flow problems: capacity, sinks and source                                                                                              |  |  |  |  |   |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | solution of small-scale network flow problems by inspection and the use of the 'maximum-flow minimum-cut' theorem to aid the solution of larger scale problems. |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | determination of the shortest path between two specified vertices in a graph, digraph or network by inspection                                                  |  |  |  |  | √ |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision | Dijkstra's algorithm and its use to determine the shortest path between a given vertex and each of the other vertices in a weighted graph or network.           |  |  |  |  |   |  |

|                          |                                 |                                                                                                                                                        |   |  |  |  |  |  |
|--------------------------|---------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|---|--|--|--|--|--|
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision        | use of a bipartite graph and its tabular or matrix form to represent a matching problem                                                                |   |  |  |  |  |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision        | determination of the optimum assignment/s of people or machines to tasks by inspection or by use of the hungarian algorithm for larger scale problems. |   |  |  |  |  |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision        | construction of an activity network from a precedence table (or equivalent) including the use of dummy activities where necessary                      |   |  |  |  |  |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision        | use of forward and backward scanning to determine the earliest starting times (EST) and latest starting times (LST) for each activity                  |   |  |  |  |  |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision        | use of ESTs and LSTs to identify the critical path in the network and determine the float times for non-critical activities                            |   |  |  |  |  |  |
| VCE<br>Further<br>Unit 4 | Networks and<br>Decision        | use of crashing to reduce the completion time of the project or task being modelled                                                                    |   |  |  |  |  |  |
| VCE<br>Further<br>Unit 4 | Geometry<br>and<br>Trigonometry | calculation of surface area and volume of spheres, cylinders, cones, pyramids and prisms, and their composites                                         | √ |  |  |  |  |  |
| VCE<br>Further<br>Unit 4 | Geometry<br>and<br>Trigonometry | application of linear scale factor $k > 0$ of similar figures and shapes to scale lengths, areas and volumes with practical applications               |   |  |  |  |  |  |



|                          |                                 |                                                                                                                                                                                                                                                                                 |  |                                  |                                  |   |                   |  |
|--------------------------|---------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|----------------------------------|----------------------------------|---|-------------------|--|
| VCE<br>Further<br>Unit 4 | Geometry<br>and<br>Trigonometry | review of the methods for solving right and non-right-angled triangles, including the ambiguous case of the sine rule, and their application to solving practical problems in two and three dimensions                                                                          |  | ✓                                | ✓                                | ✓ | ✓                 |  |
| VCE<br>Further<br>Unit 4 | Geometry<br>and<br>Trigonometry | specification of location (distance and direction) in two dimensions using three-figure bearings with applications such as navigation and orienteering, including situations involving the solution of non-right-angled triangles.                                              |  | ✓                                | ✓                                | ✓ | ✓                 |  |
| VCE<br>Further<br>Unit 4 | Geometry<br>and<br>Trigonometry | circle mensuration; arc length using the rule $180 s \approx r \pi \theta$ with practical applications                                                                                                                                                                          |  | including radians not on further | including radians not on further | ✓ | Including radians |  |
| VCE<br>Further<br>Unit 4 | Geometry<br>and<br>Trigonometry | arc length of a sector of a circle, and the areas of sectors and segments with practical applications                                                                                                                                                                           |  | ✓                                | ✓                                | ✓ | ✓                 |  |
| VCE<br>Further<br>Unit 4 | Geometry<br>and<br>Trigonometry | use of trigonometry and Pythagoras' theorem in two and three dimensions to solve problems involving the solution of right-angled triangles within a sphere                                                                                                                      |  | ✓                                | ✓                                | ✓ | ✓                 |  |
| VCE<br>Further<br>Unit 4 | Geometry<br>and<br>Trigonometry | use of a sphere of radius 6400 km as a model of the earth, and meridians and parallels and their use in locating points on the surface of the earth in terms of latitude and longitude (specified in decimal degrees) using the Greenwich meridian and the equator as reference |  |                                  |                                  |   |                   |  |
| VCE<br>Further<br>Unit 4 | Geometry<br>and<br>Trigonometry | use of meridians to determine the shortest distance from any point on the earth to a pole or the equator                                                                                                                                                                        |  |                                  |                                  |   |                   |  |

|                          |                                 |                                                                                                                                                                                                                                                                                                                         |  |                                  |                                  |                                  |                                  |  |
|--------------------------|---------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|----------------------------------|----------------------------------|----------------------------------|----------------------------------|--|
| VCE<br>Further<br>Unit 4 | Geometry<br>and<br>Trigonometry | use of a great circle to determine the shortest distance between two points on the surface of the earth that have the same longitude                                                                                                                                                                                    |  |                                  |                                  |                                  |                                  |  |
| VCE<br>Further<br>Unit 4 | Geometry<br>and<br>Trigonometry | use of 15° of longitude as equating to a 1 hour time difference to identify time zones, and determining times of journeys that cross two or more time zones from departure and arrival times.                                                                                                                           |  |                                  |                                  |                                  |                                  |  |
| VCE<br>Further<br>Unit 4 | Graphs and<br>Relations         | straight-line graphs, line segment graphs and step graphs and their use to model and analyse practical situations                                                                                                                                                                                                       |  | √                                | √                                | √                                | √                                |  |
| VCE<br>Further<br>Unit 4 | Graphs and<br>Relations         | simultaneous linear equations in two unknowns and their use to model and analyse practical situations including break-even analysis, where cost and revenue functions are linear                                                                                                                                        |  | √*Not<br>including<br>break even | √*Not<br>including<br>break even | √*Not<br>including<br>break even | √*Not<br>including<br>break even |  |
| VCE<br>Further<br>Unit 4 | Graphs and<br>Relations         | non-linear graphs and their use to model and analyse practical and familiar situations including the practical significance and interpretation of intercepts, slope, maximum/minimum points and the average rate of change when interpreting the graph                                                                  |  | √                                | √                                | √                                | √                                |  |
| VCE<br>Further<br>Unit 4 | Graphs and<br>Relations         | non-linear graphs, either constructed from a table of data or given, the use of interpolation and extrapolation to predict values, estimation of maximum/minimum values and location; and coordinates of points of intersection for applications such as break-even analysis with non-linear cost and revenue functions |  |                                  |                                  |                                  |                                  |  |

|                          |                         |                                                                                                                                                                                                                                                                                                                                                                                                                         |   |  |  |   |   |  |
|--------------------------|-------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|--|--|---|---|--|
| VCE<br>Further<br>Unit 4 | Graphs and<br>Relations | graphical representation of relations of the form of $y = kx^n$<br>for $x \geq 0$ , where $n \in \{-2, -1, 1, 2, 3\}$ , and their use in modelling practical situations including the determination of the constant of proportionality $k$ by substitution<br>of known values or by plotting $y$ against $x^n$ to linearise a given set of data, and the use of linearisation to test the validity of a proposed model. |   |  |  | √ | √ |  |
| VCE<br>Further<br>Unit 4 | Graphs and<br>Relations | review of linear inequalities in one and two variables and their graphical representation                                                                                                                                                                                                                                                                                                                               | √ |  |  |   |   |  |
| VCE<br>Further<br>Unit 4 | Graphs and<br>Relations | graphs of systems of linear inequalities (no more than five including those involving only one variable) and the use of shading-in to identify a feasible region                                                                                                                                                                                                                                                        | √ |  |  |   |   |  |
| VCE<br>Further<br>Unit 4 | Graphs and<br>Relations | linear programming and its purpose                                                                                                                                                                                                                                                                                                                                                                                      |   |  |  |   |   |  |
| VCE<br>Further<br>Unit 4 | Graphs and<br>Relations | formulation of a linear programming problem including the identification of the decision variables, the construction of a system of linear inequalities to represent the constraints, and the expression of the quantity to be optimised (the objective function) in terms of the decision variables                                                                                                                    |   |  |  |   |   |  |

## VCE Mathematical Methods

| Unit    | Content                                                                                                                                                                                                                                                                                                                          | Included in Prior learning | Included in Analysis SL | Included in Analysis HL | Included in Apps SL | Included in Apps HL | Comments        |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|-------------------------|-------------------------|---------------------|---------------------|-----------------|
| 1 and 2 | eg. the five-number summary and the boxplot as its graphical representation and display, including the use of the lower fence ( $Q1 - 1.5 \times IQR$ ) and upper fence ( $Q3 + 1.5 \times IQR$ ) to identify possible outliers                                                                                                  |                            | ✓                       | ✓                       | ✓                   | ✓                   |                 |
| 1 and 2 | <ul style="list-style-type: none"> <li>review of coordinate geometry</li> </ul>                                                                                                                                                                                                                                                  | ✓                          |                         |                         |                     |                     |                 |
| 1 and 2 | <ul style="list-style-type: none"> <li>functions and function notation, domain, co-domain and range, representation of a function by rule, graph and table</li> </ul>                                                                                                                                                            |                            | ✓                       | ✓                       | ✓                   | ✓                   |                 |
| 1 and 2 | <ul style="list-style-type: none"> <li>use of the vertical line test to determine whether a relation is a function or not, including examples of relations that are not functions and their graphs such as <math>x = k</math>, <math>x = ay^2</math> and circles in the form <math>(x - h)^2 + (y - k)^2 = r^2</math></li> </ul> |                            | ✓                       | ✓                       | ✓                   | ✓                   |                 |
| 1 and 2 | <ul style="list-style-type: none"> <li>qualitative interpretation of features of graphs of functions, including those of real data not explicitly represented by a rule, with approximate location of stationary points</li> </ul>                                                                                               |                            | ✓                       | ✓                       | ✓                   | ✓                   |                 |
| 1 and 2 | <ul style="list-style-type: none"> <li>graphs of power functions <math>f(x) = x^n</math> for <math>n \in N</math> and <math>n \in \{-2, -1\}</math>,</li> </ul>                                                                                                                                                                  |                            | ✓                       | ✓                       | ✓                   | ✓                   |                 |
| 1 and 2 | and transformations of these graphs to the form $y = a(x + b)^n + c$ where $a, b, c \in R$ and $a \neq 0$                                                                                                                                                                                                                        |                            | *                       | *                       |                     |                     | Quadratics only |
| 1 and 2 | <ul style="list-style-type: none"> <li>graphs of polynomial functions to degree 4 and other polynomials of higher degree such as <math>g(x) = (x + 2)^2(x - 1)^3 + 10</math></li> </ul>                                                                                                                                          |                            | ✓                       | ✓                       | *                   | *                   | Al - up to n=3  |
| 1 and 2 | <ul style="list-style-type: none"> <li>graphs of inverse functions.</li> </ul>                                                                                                                                                                                                                                                   |                            | ✓                       | ✓                       | ✓                   | ✓                   |                 |
|         |                                                                                                                                                                                                                                                                                                                                  |                            |                         |                         |                     |                     |                 |
| 1 and 2 | <ul style="list-style-type: none"> <li>use of symbolic notation to develop algebraic expressions and represent functions, relations, equations and systems of simultaneous equations</li> </ul>                                                                                                                                  |                            | ✓                       | ✓                       | ✓                   | ✓                   |                 |
| 1 and 2 | <ul style="list-style-type: none"> <li>substitution into and manipulation of these expressions</li> </ul>                                                                                                                                                                                                                        | ✓                          |                         |                         |                     |                     |                 |

|         |                                                                                                                                                                                                                                                                                                                                                                                                                              |  |   |   |   |   |                   |
|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|-------------------|
| 1 and 2 | <ul style="list-style-type: none"> <li>recognition of equivalent expressions and simplification of algebraic expressions involving different forms of polynomial and power functions, the use of distributive and exponent laws applied to these functions, and manipulation from one form of expression to an equivalent form, including expansion of <math>(x + a)^n</math> where <math>n \in \mathbb{N}</math></li> </ul> |  | ✓ | ✓ |   | ✓ |                   |
| 1 and 2 | <ul style="list-style-type: none"> <li>use of parameters to represent families of functions and determination of rules of simple functions and relations from given information</li> </ul>                                                                                                                                                                                                                                   |  | ✓ | ✓ | ✓ | ✓ |                   |
| 1 and 2 | <ul style="list-style-type: none"> <li>transformations of the plane and application to basic functions and relations by simple combinations of dilations (students should be familiar with both 'parallel to an axis' and 'from an axis' descriptions), reflections in an axis and translations, including the use of matrices for transformations</li> </ul>                                                                |  | ✓ | ✓ |   | ✓ | No matrices in AA |
| 1 and 2 | <ul style="list-style-type: none"> <li>the connection between the roots of a polynomial function, its factors and the horizontal axis intercepts of its graph, including the remainder, factor and rational root theorems</li> </ul>                                                                                                                                                                                         |  | ✓ | ✓ | ✓ | ✓ |                   |
| 1 and 2 | <ul style="list-style-type: none"> <li>solution of polynomial equations of low degree, numerically (including numerical approximation of roots of simple polynomial functions using bisection), graphically and algebraically</li> </ul>                                                                                                                                                                                     |  | ✓ | ✓ | ✓ | ✓ |                   |
| 1 and 2 | solution of a set of simultaneous linear equations (geometric interpretation only required for two variables) and equations of the form $f(x) = g(x)$ numerically, graphically and algebraically.                                                                                                                                                                                                                            |  |   |   |   |   |                   |
|         |                                                                                                                                                                                                                                                                                                                                                                                                                              |  | ✓ | ✓ | ✓ | ✓ |                   |
| 1 and 2 | <ul style="list-style-type: none"> <li>average and instantaneous rates of change in a variety of practical contexts and informal treatment of instantaneous rate of change as a limiting case of the average rate of change</li> </ul>                                                                                                                                                                                       |  | ✓ | ✓ | ✓ | ✓ |                   |
| 1 and 2 | <ul style="list-style-type: none"> <li>interpretation of graphs of empirical data with respect to rate of change such as temperature or pollution levels over time, motion graphs and the height of water in containers of different shapes that are being filled at a constant rate, with informal consideration of continuity and smoothness</li> </ul>                                                                    |  | ✓ | ✓ | ✓ | ✓ |                   |
| 1 and 2 | <ul style="list-style-type: none"> <li>use of gradient of a tangent at a point on the graph of a function to describe and measure instantaneous rate of change of the function, including consideration of where the rate of change is positive, negative, or zero, and the relationship of the gradient function to features of the graph of the original function.</li> </ul>                                              |  | ✓ | ✓ | ✓ | ✓ |                   |
| 1 and 2 |                                                                                                                                                                                                                                                                                                                                                                                                                              |  |   |   |   |   |                   |

|         |                                                                                                                                                                                                                                                                                             |                                            |          |   |   |   |  |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------|----------|---|---|---|--|
| 1 and 2 | <ul style="list-style-type: none"> <li>random experiments, sample spaces, outcomes, elementary and compound events</li> </ul>                                                                                                                                                               | X                                          |          |   |   |   |  |
| 1 and 2 | <ul style="list-style-type: none"> <li>simulation using simple random generators such as coins, dice, spinners and pseudo-random generators using technology, and the display and interpretation of results, including informal consideration of proportions in samples</li> </ul>          |                                            | ✓        | ✓ | ✓ | ✓ |  |
| 1 and 2 | <ul style="list-style-type: none"> <li>probability of elementary and compound events and their representation as lists, grids, venn diagrams, karnaugh maps, tables and tree diagrams</li> </ul>                                                                                            |                                            | ✓        | ✓ | ✓ | ✓ |  |
| 1 and 2 | <ul style="list-style-type: none"> <li>the addition rule for probabilities, <math>\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)</math>, and the relation that for mutually exclusive events <math>\Pr(A \cap B) = 0</math>, hence <math>\Pr(A \cup B) = \Pr(A) + \Pr(B)</math></li> </ul> |                                            | ✓        | ✓ | ✓ | ✓ |  |
|         | $\Pr(A \cap B)$                                                                                                                                                                                                                                                                             |                                            |          |   |   |   |  |
| 1 and 2 | <ul style="list-style-type: none"> <li>conditional probability in terms of reduced sample space, the relations <math>\Pr(A   B) =</math></li> </ul>                                                                                                                                         | and                                        | ✓        | ✓ | ✓ | ✓ |  |
|         |                                                                                                                                                                                                                                                                                             | $\Pr(A \cap B) = \Pr(A   B) \times \Pr(B)$ | $\Pr(B)$ |   |   |   |  |
| 1 and 2 | <ul style="list-style-type: none"> <li>the law of total probability for two events</li> </ul>                                                                                                                                                                                               |                                            |          |   |   |   |  |
|         | the relations that for pairwise independent events $A$ and $B$                                                                                                                                                                                                                              |                                            | ✓        | ✓ |   |   |  |
|         |                                                                                                                                                                                                                                                                                             |                                            |          |   |   |   |  |
| 1 and 2 | <ul style="list-style-type: none"> <li>review of trigonometry (sine and cosine rules not required)</li> </ul>                                                                                                                                                                               |                                            | ✓        | ✓ | ✓ | ✓ |  |
| 1 and 2 | <ul style="list-style-type: none"> <li>the unit circle, radians, arc length and conversion between radian and degree measures of angle</li> </ul>                                                                                                                                           |                                            | ✓        | ✓ |   |   |  |
| 1 and 2 | <ul style="list-style-type: none"> <li>sine, cosine and tangent as functions of a real variable, and the relationships <math>\sin(x) \approx x</math> for small values of <math>x</math>, <math>\sin^2(x) + \cos^2(x) = 1</math> and <math>\tan(x) =</math></li> </ul>                      |                                            | ✓        | ✓ |   |   |  |
|         |                                                                                                                                                                                                                                                                                             |                                            |          |   |   |   |  |
|         | •                                                                                                                                                                                                                                                                                           |                                            |          |   |   |   |  |
| 1 and 2 | <ul style="list-style-type: none"> <li>symmetry properties, complementary relations and periodicity properties for sine, cosine and tangent functions</li> </ul>                                                                                                                            |                                            | ✓        | ✓ |   |   |  |

|         |                                                                                                                                                                                                                                                                                                                                                  |  |   |   |   |   |             |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|-------------|
| 1 and 2 | <ul style="list-style-type: none"> <li>circular functions of the form <math>y = af(bx) + c</math> and their graphs, where <math>f</math> is the sine, cosine or tangent function, and <math>a, b, c \in R</math> with <math>a, b \neq 0</math></li> </ul>                                                                                        |  | ✓ | ✓ | * | ✓ | not tangent |
| 1 and 2 | <ul style="list-style-type: none"> <li>simple applications of sine and cosine functions of the above form, with examples from various modelling contexts, the interpretation of period, amplitude and mean value in these contexts and their relationship to the parameters <math>a, b</math> and <math>c</math></li> </ul>                      |  | ✓ | ✓ | ✓ | ✓ |             |
| 1 and 2 | <ul style="list-style-type: none"> <li>exponential functions of the form <math>f: R \rightarrow R, f(x) = Aa^{kx} + C</math> and their graphs, where <math>a \in R^+, A, k, C \in R, A \neq 0</math></li> </ul>                                                                                                                                  |  | ✓ | ✓ | ✓ | ✓ |             |
| 1 and 2 | <ul style="list-style-type: none"> <li>logarithmic functions of the form <math>f: R^+ \rightarrow R, f(x) = \log_a(x)</math>, where <math>a &gt; 1</math>, and their graphs, as the inverse function of <math>y = a^x</math>, including the relationships <math>a^{\log_a(x)} = x</math> and <math>\log_a(a^x) = x</math></li> </ul>             |  | ✓ | ✓ |   | ✓ |             |
| 1 and 2 | simple applications of exponential functions of the above form, with examples from various modelling contexts, and the interpretation of initial value, rate of growth or decay, half-life and long run value in these contexts and their relationship to the parameters $A, k$ and $C$ .                                                        |  | ✓ | ✓ | ✓ | ✓ |             |
|         |                                                                                                                                                                                                                                                                                                                                                  |  |   |   |   |   |             |
| 1 and 2 | <ul style="list-style-type: none"> <li>use of inverse functions and transformations to solve equations of the form <math>Af(bx) + c = k</math>, where <math>A, b, c, k \in R</math> and <math>A, b \neq 0</math> and <math>f</math> is sine, cosine, tangent or <math>a^x</math>, using exact or approximate values on a given domain</li> </ul> |  | ✓ | ✓ | ✓ | ✓ |             |
| 1 and 2 | <ul style="list-style-type: none"> <li>index (exponent) laws and logarithm laws, including their application to the solution of simple exponential equations</li> </ul>                                                                                                                                                                          |  | ✓ | ✓ | ✓ | ✓ |             |
| 1 and 2 | numerical approximation of roots of cubic polynomial functions using Newton's method.                                                                                                                                                                                                                                                            |  |   |   |   |   |             |
|         |                                                                                                                                                                                                                                                                                                                                                  |  |   |   |   |   |             |
| 1 and 2 | <ul style="list-style-type: none"> <li>graphical and numerical approaches to approximating the value of the gradient function for simple polynomial functions and power functions at points in the domain of the function</li> </ul>                                                                                                             |  | ✓ | ✓ | ✓ | ✓ |             |
| 1 and 2 | <ul style="list-style-type: none"> <li>the derivative as the gradient of the graph of a function at a point and its representation by a gradient function</li> </ul>                                                                                                                                                                             |  | ✓ | ✓ | ✓ | ✓ |             |
| 1 and 2 | <ul style="list-style-type: none"> <li>notations for the derivative of a function: <math>f'(x)</math></li> </ul>                                                                                                                                                                                                                                 |  | ✓ | ✓ | ✓ | ✓ |             |
|         | $\frac{dy}{dx}, \frac{d}{dx}(f(x)), D_x(f)$                                                                                                                                                                                                                                                                                                      |  |   |   |   |   |             |
|         |                                                                                                                                                                                                                                                                                                                                                  |  |   |   |   |   |             |

|         |                                                                                                                                                                                                                                                                                                                                                  |  |   |   |   |   |                                                   |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|---------------------------------------------------|
| 1 and 2 | <ul style="list-style-type: none"> <li>first principles approach to differentiation of <math>f(x) = x^n</math>, <math>n \in \mathbb{Z}</math>, and simple polynomial functions</li> </ul>                                                                                                                                                        |  |   | ✓ |   |   |                                                   |
| 1 and 2 | <ul style="list-style-type: none"> <li>derivatives of simple power functions and polynomial functions by rule</li> </ul>                                                                                                                                                                                                                         |  | ✓ | ✓ | ✓ | ✓ |                                                   |
| 1 and 2 | <ul style="list-style-type: none"> <li>applications of differentiation, including finding instantaneous rates of change, stationary values of functions, local maxima or minima, points of inflection, analysing graphs of functions, solving maximum and minimum problems and solving simple problems involving straight-line motion</li> </ul> |  | ✓ | ✓ | * | * | Using stationary points for optimisation problems |
| 1 and 2 | <ul style="list-style-type: none"> <li>notations for an anti-derivative, primitive or indefinite integral of a function: <math>F(x)</math>, <math>\int f(x) dx</math></li> </ul>                                                                                                                                                                 |  | ✓ | ✓ | ✓ | ✓ |                                                   |
| 1 and 2 | <ul style="list-style-type: none"> <li>use of a boundary condition to determine a specific anti-derivative of a given function</li> </ul>                                                                                                                                                                                                        |  | ✓ | ✓ | ✓ | ✓ |                                                   |
| 1 and 2 | anti-differentiation as the inverse process of differentiation and identification of families of curves with the same gradient function, including application of anti-differentiation to solving simple problems involving straight-line motion.                                                                                                |  | ✓ | ✓ |   | ✓ |                                                   |
|         |                                                                                                                                                                                                                                                                                                                                                  |  |   |   |   |   |                                                   |
| 1 and 2 | <ul style="list-style-type: none"> <li>addition and multiplication principles for counting</li> </ul>                                                                                                                                                                                                                                            |  | ✓ | ✓ |   |   |                                                   |
| 1 and 2 | combinations: concept of a selection and computation of ${}^nC_r$ , application of counting techniques to probability.                                                                                                                                                                                                                           |  | ✓ | ✓ |   |   |                                                   |
|         |                                                                                                                                                                                                                                                                                                                                                  |  |   |   |   |   |                                                   |
| 3 and 4 | <ul style="list-style-type: none"> <li>graphs and identification of key features of graphs of the following functions:</li> </ul>                                                                                                                                                                                                                |  |   |   |   |   |                                                   |
| 3 and 4 | – power functions, $y = x^n$ , $n \in \mathbb{Q}$                                                                                                                                                                                                                                                                                                |  | ✓ | ✓ | * | * | With tech                                         |
| 3 and 4 | – exponential functions, $y = a^x$ , $a \in \mathbb{R}^+$ , in particular $y = e^x$ , and logarithmic functions, $y = \log_e(x)$ and $y = \log_{10}(x)$                                                                                                                                                                                          |  | ✓ | ✓ | * | * | With tech                                         |
| 3 and 4 | – circular functions, $y = \sin(x)$ , $y = \cos(x)$ and $y = \tan(x)$                                                                                                                                                                                                                                                                            |  | ✓ | ✓ | * | * | With tech                                         |
| 3 and 4 | <ul style="list-style-type: none"> <li>graphs of polynomial functions</li> </ul>                                                                                                                                                                                                                                                                 |  | ✓ | ✓ | * | * | Up to cubics with tech                            |
| 3 and 4 | <ul style="list-style-type: none"> <li>transformation from <math>y = f(x)</math> to <math>y = Af(n(x+b)) + c</math>, where <math>A</math>, <math>n</math>, <math>b</math> and <math>c \in \mathbb{R}</math>, <math>A, n \neq 0</math>, and <math>f</math> is one of the functions specified above, and the inverse transformation</li> </ul>     |  | ✓ | ✓ | ✓ | ✓ |                                                   |
| 3 and 4 | the relation between the graph of an original function and the graph of a corresponding transformed function (including families of transformed functions for a single transformation parameter)                                                                                                                                                 |  | ✓ | ✓ | ✓ | ✓ |                                                   |



|         |                                                                                                                                                                                                                                                                                                                                                                                                                                 |  |   |   |   |   |                            |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|----------------------------|
| 3 and 4 | graphs of sum, difference, product and composite functions where $f$ and $g$ are functions of the types specified above (not including composite functions that result in reciprocal or quotient functions), use of polynomial, power, circular, exponential and logarithmic functions, simple transformation and combinations of these functions, including simple piecewise (hybrid) functions, to model practical situations |  | * | * | * | * | With tech                  |
| 3 and 4 | • review of algebra of polynomials, equating coefficients and solution of polynomial equations with real coefficients of degree $n$ having up to $n$ real solutions                                                                                                                                                                                                                                                             |  |   |   |   |   |                            |
| 3 and 4 | • use of simple functional relations such as $f(x+k) = f(x)$ , $f(x^n) = nf(x)$ , $f(x) + f(-x) = 0$ , $f(xy) = f(x)f(y)$ , to characterise properties of functions including periodicity and symmetry, and to specify algebraic equivalence, including the exponent and logarithm laws                                                                                                                                         |  | ✓ | ✓ |   | ✓ |                            |
| 3 and 4 | • functions and their inverses, including conditions for the existence of an inverse function, and use of inverse functions to solve equations involving exponential, logarithmic, circular and power functions                                                                                                                                                                                                                 |  | ✓ | ✓ | * | ✓ | Not with restricted domain |
| 3 and 4 | • composition of functions, where $f$ composition $g$ is defined by $f(g(x))$ , given $r_g \subseteq d_f$ (the notation $f \circ g$ may be used, but is not required)                                                                                                                                                                                                                                                           |  | ✓ | ✓ |   | ✓ |                            |
| 3 and 4 | • solution of equations of the form $f(x) = g(x)$ over a specified interval, where $f$ and $g$ are functions of the type specified in the 'Functions and graphs' area of study, by graphical, numerical and algebraic methods, as applicable                                                                                                                                                                                    |  | * | * | * | * | With tech                  |
| 3 and 4 | • solution of literal equations and general solution of equations involving a single parameter                                                                                                                                                                                                                                                                                                                                  |  |   |   |   |   |                            |
| 3 and 4 | solution of simple systems of simultaneous linear equations, including consideration of cases where no solution or an infinite number of possible solutions exist (geometric interpretation only required for two equations in two variables).                                                                                                                                                                                  |  | ✓ | ✓ | * | * | With tech                  |
| 3 and 4 | • review of average and instantaneous rates of change, tangents to the graph of a given function and the derivative function                                                                                                                                                                                                                                                                                                    |  | ✓ | ✓ | ✓ | ✓ |                            |
| 3 and 4 | • deducing the graph of the derivative function from the graph of a given function and deducing the graph of an anti-derivative function from the graph of a given function                                                                                                                                                                                                                                                     |  | ✓ | ✓ |   |   |                            |
| 3 and 4 | derivatives of $x^n$ , for $n \in \mathbb{Q}$ , $e^x$ , $\log_e(x)$ , $\sin(x)$ , $\cos(x)$ and $\tan(x)$                                                                                                                                                                                                                                                                                                                       |  | ✓ | ✓ | * | ✓ | $x^n$ only                 |

|         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |   |   |   |   |           |
|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|-----------|
| 3 and 4 | <ul style="list-style-type: none"> <li>derivatives of <math>f(x) \pm g(x)</math>, <math>f(x) \times g(x)</math>,</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |  | ✓ | ✓ |   | ✓ |           |
|         | $f(x)$ and $f(g(x))$ where $f$ and $g$ are polynomial functions, exponential, $g(x)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |  |   |   |   |   |           |
| 3 and 4 | circular, logarithmic or power functions and transformations or simple combinations of these functions                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |  | ✓ | ✓ |   | ✓ |           |
| 3 and 4 | <ul style="list-style-type: none"> <li>application of differentiation to graph sketching and identification of key features of graphs, identification of intervals over which a function is constant, stationary, strictly increasing or strictly decreasing, identification of the maximum rate of increase or decrease in a given application context (consideration of the second derivative is not required), identification of local maximum/minimum values over an interval and application to solving problems, and identification of interval endpoint maximum and minimum values</li> </ul> |  | ✓ | ✓ | ✓ | ✓ |           |
| 3 and 4 | <ul style="list-style-type: none"> <li>anti-derivatives of polynomial functions and functions of the form <math>f(ax + b)</math> where <math>f</math> is <math>x^n</math>, for <math>n \in \mathbb{Q}</math>, <math>e^x</math>, <math>\sin(x)</math>, <math>\cos(x)</math> and linear combinations of these</li> </ul>                                                                                                                                                                                                                                                                               |  | ✓ | ✓ |   | ✓ |           |
| 3 and 4 | <ul style="list-style-type: none"> <li>informal consideration of the definite integral as a limiting value of a sum involving quantities such as area under a curve, including examples such as distance travelled in a straight line and cumulative effects of growth such as inflation</li> </ul>                                                                                                                                                                                                                                                                                                  |  | ✓ | ✓ | * | ✓ | With tech |
| 3 and 4 | <ul style="list-style-type: none"> <li>anti-differentiation by recognition that <math>F'(x) = f(x)</math> implies <math>\int f(x) dx = F(x) + c</math></li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                    |  | ✓ | ✓ |   | ✓ |           |
|         | $b$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |   |   |   |   |           |
| 3 and 4 | <ul style="list-style-type: none"> <li>informal treatment of the fundamental theorem of calculus, <math>\int_a^b f(x) dx = F(b) - F(a)</math></li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                             |  | ✓ | ✓ |   |   |           |
|         | $a$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |   |   |   |   |           |
| 3 and 4 | <ul style="list-style-type: none"> <li>properties of anti-derivatives and definite integrals</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |  |   |   |   |   |           |
| 3 and 4 | application of integration to problems involving finding a function from a known rate of change given a boundary condition, calculation of the area of a region under a curve and simple cases of areas between curves, distance travelled in a straight line, average value of a function and other situations.                                                                                                                                                                                                                                                                                     |  | ✓ | ✓ | * | ✓ |           |
|         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |   |   |   |   |           |

|         |                                                                                                                                                                                                                      |  |   |   |   |   |           |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|-----------|
| 3 and 4 | <ul style="list-style-type: none"> <li>random variables, including the concept of a random variable as a real function defined on a sample space and examples of discrete and continuous random variables</li> </ul> |  | ✓ | ✓ | ✓ | ✓ |           |
| 3 and 4 | <ul style="list-style-type: none"> <li>discrete random variables:</li> </ul>                                                                                                                                         |  |   |   |   |   |           |
| 3 and 4 | – specification of probability distributions for discrete random variables using graphs, tables and probability mass functions                                                                                       |  | ✓ | ✓ | ✓ | ✓ |           |
| 3 and 4 | – calculation and interpretation and use of mean ( $\mu$ ), variance ( $\sigma^2$ ) and standard deviation of a discrete random variable and their use                                                               |  | ✓ | ✓ | * | * | With tech |
| 3 and 4 | – bernoulli trials and the binomial distribution, $Bi(n, p)$ , as an example of a probability distribution for a discrete random variable                                                                            |  | ✓ | ✓ | ✓ | ✓ |           |
| 3 and 4 | – effect of variation in the value/s of defining parameters on the graph of a given probability mass function for a discrete random variable                                                                         |  | ✓ | ✓ |   |   |           |
| 3 and 4 | – calculation of probabilities for specific values of a random variable and intervals defined in terms of a random variable, including conditional probability                                                       |  | ✓ | ✓ | ✓ | ✓ |           |
| 3 and 4 | continuous random variables:                                                                                                                                                                                         |  |   |   |   |   |           |
| 3 and 4 | – construction of probability density functions from non-negative functions of a real variable                                                                                                                       |  |   | ✓ |   |   |           |
| 3 and 4 | – specification of probability distributions for continuous random variables using probability density functions                                                                                                     |  |   | ✓ |   |   |           |
| 3 and 4 | – calculation and interpretation of mean ( $\mu$ ), median, variance ( $\sigma^2$ ) and standard deviation of a continuous random variable and their use                                                             |  |   | ✓ |   |   |           |
| 3 and 4 | – standard normal distribution, $N(0, 1)$ , and transformed normal distributions, $N(\mu, \sigma^2)$ , as examples of a probability distribution for a continuous random variable                                    |  | ✓ | ✓ | ✓ | ✓ |           |
| 3 and 4 | – effect of variation in the value/s of defining parameters on the graph of a given probability density function for a continuous random variable                                                                    |  | ✓ | ✓ | ✓ | ✓ |           |
| 3 and 4 | – calculation of probabilities for intervals defined in terms of a random variable, including conditional probability (the cumulative distribution function may be used but is not required)                         |  | ✓ | ✓ | ✓ | ✓ |           |
| 3 and 4 | <ul style="list-style-type: none"> <li>Statistical inference, including definition and distribution of sample proportions, simulations and confidence intervals:</li> </ul>                                          |  |   |   |   | ✓ |           |

|         |                                                                                                                                                                                                                                                                           |  |  |  |  |   |  |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|--|---|--|
| 3 and 4 | – distinction between a population <i>parameter</i> and a sample <i>statistic</i> and the use of the sample statistic to estimate the population parameter                                                                                                                |  |  |  |  | √ |  |
| 3 and 4 | – concept of the sample proportion $\hat{P} = \frac{X}{n}$ as a random variable whose value varies between samples, where $X$ is a binomial random variable which is associated with the number of items that have a particular characteristic and $n$ is the sample size |  |  |  |  |   |  |
| 3 and 4 | – approximate normality of the distribution of $\hat{p}$ for large samples and, for such a situation, the mean $p$ , (the population proportion) and standard deviation,                                                                                                  |  |  |  |  |   |  |
|         |                                                                                                                                                                                                                                                                           |  |  |  |  |   |  |
| 3 and 4 | – simulation of random sampling, for a variety of values of $p$ and a range of sample sizes, to illustrate the distribution of $\hat{p}$                                                                                                                                  |  |  |  |  |   |  |
| 3 and 4 | – determination of, from a large sample, an approximate confidence interval                                                                                                                                                                                               |  |  |  |  |   |  |
| 3 and 4 | $\hat{p} \pm z$ for a population proportion where $z$ is the appropriate quantile for the                                                                                                                                                                                 |  |  |  |  |   |  |
| 3 and 4 | standard normal distribution, in particular the 95% confidence interval as an example of such an interval where $z \approx 1.96$ (the term standard error may be used but is not required).                                                                               |  |  |  |  | √ |  |

## VCE Specialist Mathematics

| Course                       | Unit    | Content                                                                                                                                                                                                                                    | Included in Prior learning | Included in Analysis SL | Included in Analysis HL | Included in Apps SL | Included in Apps HL   | Comments              |
|------------------------------|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|-------------------------|-------------------------|---------------------|-----------------------|-----------------------|
| Arithmetic and Number        |         |                                                                                                                                                                                                                                            |                            |                         |                         |                     |                       |                       |
| Number systems and recursion |         |                                                                                                                                                                                                                                            |                            |                         |                         |                     |                       |                       |
| Specialist                   | 1 and 2 | definition and properties of the natural numbers $\mathbb{N}$ , arithmetic, order, primes, divisibility and related proofs, including the infinitude of primes                                                                             | $\sqrt{*}$                 |                         | AHL1.15                 |                     |                       | *everything but proof |
| Specialist                   | 1 and 2 | definition and properties of the rational numbers $\mathbb{Q}$ , arithmetic, order and the equivalence between fraction and decimal forms                                                                                                  | $\sqrt{*}$                 |                         |                         |                     |                       |                       |
| Specialist                   | 1 and 2 | sequences and series as maps between the natural numbers and the real numbers, the use of technology to generate sequences and series and their graphs, and sequences generated by recursion, including arithmetic and geometric sequences |                            | SL1.2-4                 | SL1.2-4                 | SL1.2-4             | SL1.2-4               |                       |
| Specialist                   | 1 and 2 | proof by mathematical induction, for example, the Tower of Hanoi, formula for the sum of the first $n$ square numbers                                                                                                                      |                            |                         | AHL1.14                 |                     |                       |                       |
| Specialist                   | 1 and 2 | limiting behaviour as $n \rightarrow \infty$ of the terms $t_n$ in a geometric sequence, the sum of the first $n$ terms $S_n$ and their dependence on the value of the common ratio $r$                                                    |                            | SL1.3, SL1.4, SL1.8     | SL1.3, SL1.4, SL1.8     | SL1.3, SL1.4        | SL1.3, SL1.4, AHL1.11 |                       |
| Specialist                   | 1 and 2 | definition and properties of the real numbers including the absolute value of a real number and their one-to-one correspondence with points on a line to produce the real number line                                                      | $\sqrt{}$                  |                         |                         |                     |                       |                       |
| Specialist                   | 1 and 2 | proof of irrationality for some real numbers such as surds of the form $\sqrt{n}$ where $n$ is not a perfect square, the golden ratio $\phi$ and logarithms such as $\log_2(5)$                                                            |                            |                         | AHL1.15                 |                     |                       |                       |

|                                        |         |                                                                                                                                                                                                                                                                                                             |             |        |                    |        |         |                                                          |
|----------------------------------------|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|--------|--------------------|--------|---------|----------------------------------------------------------|
| Specialist                             | 1 and 2 | definition and properties of the complex numbers $\mathbb{C}$ , arithmetic, modulus of a complex number, the representation of complex numbers as points on an argand diagram, general solution of quadratic equations, with real coefficients, of a single variable over $\mathbb{C}$ and conjugate roots. |             |        | AHL1.12<br>AHL1.14 |        | AHL1.12 | *no complex conjugate roots                              |
| Geometry, Measurement and Trigonometry |         |                                                                                                                                                                                                                                                                                                             |             |        |                    |        |         |                                                          |
| Geometry in the plane and proof        |         |                                                                                                                                                                                                                                                                                                             |             |        |                    |        |         |                                                          |
| Specialist                             | 1 and 2 | geometric objects and relations: point, line, parallel, perpendicular, plane, angle, polygons, circles and semi- circles, arcs, chords, segments, sectors, secants, tangents, similarity and congruence                                                                                                     | $\sqrt{}$ * |        |                    |        |         | *not similarity and congruence                           |
| Specialist                             | 1 and 2 | straight edge and compass and dynamic geometry construction of these objects and illustration of these relations, including exact angles multiples of $30^\circ$ and $45^\circ$                                                                                                                             |             |        |                    |        |         |                                                          |
| Specialist                             | 1 and 2 | principles of proof including propositions and quantifiers, examples and counter-examples, direct proof, proof by contradiction, and proof using contrapositive; and the role of diagrams in geometric proof                                                                                                |             | SL1.6* | SL1.6*<br>AHL1.15^ |        |         | *simple deductive proofs ^not proof using contrapositive |
| Specialist                             | 1 and 2 | proofs of Pythagoras' theorem, properties of quadrilaterals, interior angles and angle sums of polygons                                                                                                                                                                                                     | $\sqrt{}$ * | SL3.3  | SL3.3              | SL3.3  | SL3.3   | *not proof                                               |
| Specialist                             | 1 and 2 | congruence of triangles and the sine and cosine rules including applications                                                                                                                                                                                                                                |             | SL3.2* | SL3.2*             | SL3.2* | SL3.2*  | *no congruence                                           |
| Specialist                             | 1 and 2 | proof of circle theorems                                                                                                                                                                                                                                                                                    |             |        |                    |        |         |                                                          |
| Vectors in the plane                   |         |                                                                                                                                                                                                                                                                                                             |             |        |                    |        |         |                                                          |
| Specialist                             | 1 and 2 | representation of plane vectors as directed lines segments, examples involving position, displacement and velocity                                                                                                                                                                                          |             |        | AHL3.12            |        | AHL3.10 |                                                          |
| Specialist                             | 1 and 2 | magnitude and direction of a plane vector, and unit vectors                                                                                                                                                                                                                                                 |             |        | AHL3.12            |        | AHL3.10 |                                                          |

|                                           |         |                                                                                                                                                                              |             |       |                        |  |                       |                                                                                |
|-------------------------------------------|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|-------|------------------------|--|-----------------------|--------------------------------------------------------------------------------|
| Specialist                                | 1 and 2 | geometric representation of addition, subtraction (triangle and/or parallelogram rules) scalar multiple and linear combination of plane vectors                              |             |       | AHL3.12                |  | AHL3.10               |                                                                                |
| Specialist                                | 1 and 2 | representation of a plane vector as an ordered pair (a, b) and as a column matrix                                                                                            |             |       | AHL3.12                |  | AHL3.10               |                                                                                |
| Specialist                                | 1 and 2 | representation of a vector (a, b) in the form $a\mathbf{i} + b\mathbf{j}$ where $\mathbf{i}$ and $\mathbf{j}$ are the standard orthogonal unit vectors and direction cosines |             |       | AHL3.12                |  | AHL3.10               |                                                                                |
| Specialist                                | 1 and 2 | simple vector algebra (addition, subtraction, multiplication by a scalar, linear combination) using these forms                                                              |             |       | AHL3.12                |  | AHL3.10               |                                                                                |
| Specialist                                | 1 and 2 | a scalar product of two plane vectors, perpendicular and parallel vectors, projection of one vector onto another, and angle between two vectors                              |             |       | AHL3.13                |  | AHL3.13               |                                                                                |
| Specialist                                | 1 and 2 | application of vectors to geometric proofs, orienteering, navigation, and statics                                                                                            |             |       |                        |  | AHL3.12*              | *kinematics only                                                               |
| Graphs of Linear and Non-linear Relations |         |                                                                                                                                                                              |             |       |                        |  |                       |                                                                                |
| Graphs of non-linear relations            |         |                                                                                                                                                                              |             |       |                        |  |                       |                                                                                |
| Specialist                                | 1 and 2 | interpreting graphical representations of data such as daily UV levels or water storage levels over time                                                                     | $\sqrt{}$ * |       |                        |  |                       |                                                                                |
| Specialist                                | 1 and 2 | graphs of simple reciprocal functions, including those for sine, cosine and tangent                                                                                          |             | SL2.8 | SL2.8, AHL2.13, AHL3.9 |  |                       |                                                                                |
| Specialist                                | 1 and 2 | locus definition and construction in the plane of lines, parabolas, circles, ellipses and hyperbolas                                                                         |             |       |                        |  |                       |                                                                                |
| Specialist                                | 1 and 2 | cartesian, polar and parametric forms and graphs of lines, parabolas, circles, ellipses and hyperbolas                                                                       |             |       | AHL1.13^<br>AHL3.14*   |  | AHL3.11^,<br>AHL3.12* | *parametric forms of lines only (vectors) ^polar form for complex numbers only |
| Specialist                                | 1 and 2 | polar form and graphs of other relations in the plane such as limaçons, cardioids, roses, lemniscates and spirals                                                            |             |       |                        |  |                       |                                                                                |
| Specialist                                | 1 and 2 | parametric form and graphs of other relations in the plane such as spirals, cycloids, lissajous figures and epicycles                                                        |             |       |                        |  |                       |                                                                                |

| Other Topics                               |         |                                                                                                                                                                                                                                                                                                                                                  |            |         |          |  |                   |                                                                                |
|--------------------------------------------|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|---------|----------|--|-------------------|--------------------------------------------------------------------------------|
| Algebra and Structure - logic and algebra  |         |                                                                                                                                                                                                                                                                                                                                                  |            |         |          |  |                   |                                                                                |
| Specialist                                 | 1 and 2 | atomic and compound propositions, connectives, truth values, Karnaugh maps and truth tables                                                                                                                                                                                                                                                      |            |         |          |  |                   |                                                                                |
| Specialist                                 | 1 and 2 | tautologies, validity and proof patterns and the application of these to proofs in natural language and in mathematics                                                                                                                                                                                                                           |            |         |          |  |                   |                                                                                |
| Specialist                                 | 1 and 2 | boolean algebra, the algebra of sets and propositional logic                                                                                                                                                                                                                                                                                     |            |         |          |  |                   |                                                                                |
| Specialist                                 | 1 and 2 | electronic gates and circuits and circuit simplification                                                                                                                                                                                                                                                                                         |            |         |          |  |                   |                                                                                |
| Specialist                                 | 1 and 2 | boolean operators and their use in search engines and databases                                                                                                                                                                                                                                                                                  |            |         |          |  |                   |                                                                                |
| Transformations, Trigonometry and Matrices |         |                                                                                                                                                                                                                                                                                                                                                  |            |         |          |  |                   |                                                                                |
| Specialist                                 | 1 and 2 | points in the plane, coordinates and their representation as $2 \times 1$ matrices (column vectors)                                                                                                                                                                                                                                              |            |         | AHL3.12  |  | AHL3.10           |                                                                                |
| Specialist                                 | 2 and 2 | linear transformations of the plane $(x, y) \rightarrow (ax + by, cx + dy)$ as a map of the plane onto itself, dilations (students should be familiar with both 'parallel to an axis' and 'from an axis' descriptions), rotations about the origin and reflection in a line through the origin and their representation as $2 \times 2$ matrices |            | SL2.11^ | SL2.11^  |  | AHL2.8*<br>AHL3.9 | *no matrices ^transformations not of form $f(ax+b)$                            |
| Specialist                                 | 3 and 2 | effect of these linear transformations and their inverse transformations, and compositions of these transformations on subsets of the plane such as points, lines, shapes and graphs                                                                                                                                                             | $\sqrt{*}$ | SL3.7^  |          |  |                   | *simple geometric ^graphs only                                                 |
| Specialist                                 | 4 and 2 | invariance of properties under transformation and the relationship between the determinant of a transformation matrix and the effect of the linear transformation on area                                                                                                                                                                        |            |         |          |  |                   |                                                                                |
| Specialist                                 | 5 and 2 | use of matrix multiplication to obtain mathematical results, such as $\sin(x + y) = \sin(x) \cos(y) + \sin(y) \cos(x)$ and the equivalence between a rotation about the origin and                                                                                                                                                               |            |         | AHL3.10* |  | AHL1.14^          | *Trig compound formula<br>^matrix multiplication and properties and a lot more |



[illegible]

|                                                        |         |                                                                                                                                                                                                                                                                            |  |       |                |  |                    |                                         |
|--------------------------------------------------------|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|-------|----------------|--|--------------------|-----------------------------------------|
| Specialist                                             | 1 and 2 | vertices and edges for undirected graphs and their representation using lists, diagrams and matrices (including multiple edges and loops) with examples from a range of contexts such as molecular structure, electrical circuits, social networks and utility connections |  |       |                |  | AHL3.14            |                                         |
| Specialist                                             | 1 and 2 | use of examples to discuss types of problems in graph theory including existence problems, construction problems, counting problems and optimisation problems                                                                                                              |  |       |                |  | AHL3.14            |                                         |
| Specialist                                             | 1 and 2 | degree of a vertex and the result that the sum of all the vertex degrees is equal to twice the number of edges (handshaking lemma)                                                                                                                                         |  |       |                |  | AHL3.14            |                                         |
| Specialist                                             | 1 and 2 | simple graphs, sub-graphs, connectedness, complete graphs and the complement of a graph, and isomorphism of graphs                                                                                                                                                         |  |       |                |  | AHL3.14            |                                         |
| Specialist                                             | 1 and 2 | bi-partite graphs, trees, regular graphs (including the platonic graphs), planar graphs and related proofs and applications                                                                                                                                                |  |       |                |  | AHL3.14*           | *trees only                             |
| Specialist                                             | 1 and 2 | walks, trails, paths, cycles and circuits, eulerian circuits and eulerian trails, hamiltonian cycles and paths                                                                                                                                                             |  |       |                |  | AHL3.15            | And some more adjacency matrices        |
| Graphs of Linear and Non-linear Relations - kinematics |         |                                                                                                                                                                                                                                                                            |  |       |                |  |                    |                                         |
| Specialist                                             | 1 and 2 | diagrammatic and graphical representation of empirical position-time data for a single particle in rectilinear motion, including examples with variable velocity                                                                                                           |  | SL5.9 | SL5.9, AHL3.14 |  | AHL3.12<br>AHL5.13 | Vector applications and differentiation |
| Specialist                                             | 1 and 2 | graphical modelling and numerical analysis of position-time and velocity-time including consideration of average velocity and distance travelled over an interval                                                                                                          |  | SL5.9 | SL5.9, AHL3.14 |  | AHL3.12<br>AHL5.13 |                                         |
| Specialist                                             | 1 and 2 | modelling and analysis of rectilinear motion under constant acceleration, including use of constant acceleration formulas                                                                                                                                                  |  | SL5.9 | SL5.9, AHL3.14 |  | AHL3.12<br>AHL5.13 |                                         |

|                                                              |         |                                                                                                                                                                                                                                                                                                          |             |       |                |       |                    |                                                                   |
|--------------------------------------------------------------|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|-------|----------------|-------|--------------------|-------------------------------------------------------------------|
| Specialist                                                   | 1 and 2 | qualitative graphical analysis of the relationship between position-time, velocity-time and acceleration-time graphs for simple cases of rectilinear motion involving variable acceleration                                                                                                              |             | SL5.9 | SL5.9, AHL3.14 |       | AHL3.12<br>AHL5.13 |                                                                   |
| Specialist                                                   | 1 and 2 | numerical approximation to instantaneous rate of change of a function $f$ at time $t = a$ by evaluation of the central difference for small values of $h$ ; and its application to approximate evaluation of instantaneous velocity and instantaneous acceleration in simple cases of rectilinear motion |             |       |                |       |                    |                                                                   |
| Specialist                                                   | 1 and 2 | approximation of velocity-time relationships by step functions; and its application to approximate evaluation of distance travelled in simple cases of rectilinear motion involving variable velocity and variable acceleration, as a sum of areas of rectangles                                         |             |       |                |       |                    |                                                                   |
| Statistics - simulation, sampling and sampling distributions |         |                                                                                                                                                                                                                                                                                                          |             |       |                |       |                    |                                                                   |
| Specialist                                                   | 1 and 2 | random experiments, events and event spaces                                                                                                                                                                                                                                                              | $\sqrt{}$ * |       |                |       |                    | *AI                                                               |
| Specialist                                                   | 1 and 2 | use of simulation to generate a random sample                                                                                                                                                                                                                                                            |             |       |                | SL4.1 | SL4.1              |                                                                   |
| Specialist                                                   | 1 and 2 | simple random sampling from a finite population and the probability of obtaining a particular sample                                                                                                                                                                                                     |             | SL4.1 | SL4.1          | SL4.1 | SL4.1              |                                                                   |
| Specialist                                                   | 1 and 2 | introduction to random variables for discrete distributions                                                                                                                                                                                                                                              |             | SL4.7 | SL4.7          | SL4.7 | SL4.7              |                                                                   |
| Specialist                                                   | 1 and 2 | distinction between a population parameter and a sample statistic and use of the sample statistics mean and proportion as an estimate of the associated population parameter mean $\mu$ and proportion $p$                                                                                               |             |       |                |       | AHL4.18*           | *Testing for population mean for Normal and Poisson distribution. |
| Specialist                                                   | 1 and 2 | concept of a sampling distribution and its random variable                                                                                                                                                                                                                                               |             |       |                |       | AHL4.18*           | *Testing for population mean for Normal and Poisson distribution  |
| Specialist                                                   | 1 and 2 | distribution of sample means and proportions considered empirically, including comparing the distributions of different size samples from the same population in terms of centre and spread                                                                                                              |             |       |                |       | AHL4.18*           | *Testing for population mean for Normal and Poisson distribution  |

|                      |         |                                                                                                                                                                                                                                                                                     |  |        |                  |  |          |                                                                  |
|----------------------|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--------|------------------|--|----------|------------------------------------------------------------------|
| Specialist           | 1 and 2 | display of variation in sample proportions and means through dot plots and other displays and considering the centre and spread of these distributions                                                                                                                              |  |        |                  |  | AHL4.18* | *Testing for population mean for Normal and Poisson distribution |
| Specialist           | 1 and 2 | consideration of the mean and standard deviation of both the distribution of sample means and the distribution of sample proportions and consideration of the effect of taking larger samples                                                                                       |  |        |                  |  | AHL4.18* | *Testing for population mean for Normal and Poisson distribution |
| Functions and Graphs |         |                                                                                                                                                                                                                                                                                     |  |        |                  |  |          |                                                                  |
| Specialist           | 3 and 4 | graphs of rational functions of low degree, their asymptotic behaviour and nature and location of stationary points                                                                                                                                                                 |  | SL2.8* | SL2.8*<br>AL2.13 |  |          | *Not stationary points                                           |
| Specialist           | 3 and 4 | absolute value function, its graph and simple transformations of the graph                                                                                                                                                                                                          |  |        | AL2.16           |  |          |                                                                  |
| Specialist           | 3 and 4 | graphs of the reciprocal circular functions cosecant, secant and cotangent, and simple transformations of these                                                                                                                                                                     |  |        | AL3.9            |  |          |                                                                  |
| Specialist           | 3 and 4 | Compound and double angle formulas for sine, cosine and tangent and the identities: $\sec^2(x) = 1 + \tan^2(x)$ and $\operatorname{cosec}^2(x) = 1 + \cot^2(x)$                                                                                                                     |  |        | AL3.10<br>AL3.9  |  |          |                                                                  |
| Specialist           | 3 and 4 | Graphs of the restricted circular functions of sine, cosine and tangent over principal domains and their respective inverse functions $\sin^{-1}$ , $\cos^{-1}$ and $\tan^{-1}$ (students should be familiar with alternative notations) and simple transformations of these graphs |  | SL3.7* | SL3.7<br>AL3.9   |  |          | *not graphs of inverse trig                                      |
| Specialist           | 3 and 4 | graphs of simple quotient functions.                                                                                                                                                                                                                                                |  | SL2.8* | SL2.8<br>AL2.13  |  |          | *linear only                                                     |
| Algebra              |         |                                                                                                                                                                                                                                                                                     |  |        |                  |  |          |                                                                  |
| Specialist           | 3 and 4 | Rational functions of a real variable, including definition of a rational function and expression of rational functions of low degree as sums of partial fractions.                                                                                                                 |  | SL2.8* | SL2.8<br>AL2.13  |  |          | *linear only                                                     |

|            |         |                                                                                                                                                                                                     |  |       |                  |  |           |                              |
|------------|---------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|-------|------------------|--|-----------|------------------------------|
| Specialist | 3 and 4 | $\mathbb{C}$ , the set of numbers $z$ of the form $z = x + yi$ where $x, y$ are real numbers and $i^2 = -1$ , real and imaginary parts, complex conjugates, modulus                                 |  |       | AL1.12<br>AL1.14 |  | ✓ AHL1.12 |                              |
| Specialist | 3 and 4 | Use of an argand diagram to represent points, lines, rays and circles in the complex plane                                                                                                          |  |       | AL1.12           |  | ✓ AHL1.12 |                              |
| Specialist | 3 and 4 | equality, addition, subtraction, multiplication and division of complex numbers                                                                                                                     |  |       | AL1.13           |  | ✓ AHL1.13 |                              |
| Specialist | 3 and 4 | polar form (modulus and argument); multiplication and division in polar form, including their geometric representation and interpretation, proof of basic identities involving modulus and argument |  |       | AL1.13           |  | ✓ AHL1.13 | basic proof of identities??? |
| Specialist | 3 and 4 | De Moivre's theorem, proof for integral powers, powers and roots of complex numbers in polar form, and their geometric representation and interpretation                                            |  |       | AL1.14           |  |           |                              |
| Specialist | 3 and 4 | $n$ th roots of unity and other complex numbers and their location in the complex plane                                                                                                             |  |       | AL1.14           |  | ✓ AHL1.12 |                              |
| Specialist | 3 and 4 | factors over $\mathbb{C}$ of polynomials with integer coefficients; and informal introduction to the fundamental theorem of algebra                                                                 |  |       | AL1.14           |  |           |                              |
| Specialist | 3 and 4 | factorisation of polynomial functions of a single variable over $\mathbb{C}$ , for example, $z^8 + 1$ , $z^2 - i$ , $z^3 - (2 - i)z^2 + z - 2 + i$                                                  |  |       |                  |  |           |                              |
| Specialist | 3 and 4 | Solution over $\mathbb{C}$ of corresponding polynomial equations by completing the square, factorisation and the conjugate root theorem.                                                            |  |       |                  |  | ✓ AHL1.12 |                              |
| Calculus   |         |                                                                                                                                                                                                     |  |       |                  |  |           |                              |
| Specialist | 3 and 4 | derivatives of inverse circular functions                                                                                                                                                           |  |       | AL5.15           |  |           |                              |
| Specialist | 3 and 4 | second derivatives, use of notations $f''(x)$ and $d^2y/dx^2$ including points of inflection and concavity and their application to the analysis of graphs of functions                             |  | SL5.7 | SL5.7            |  | ✓ AHL5.10 |                              |

|            |         |                                                                                                                                                                                                                                                                                          |  |         |                   |  |           |                                |
|------------|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------|-------------------|--|-----------|--------------------------------|
| Specialist | 3 and 4 | applications of chain rule to related rates of change and implicit differentiation; for example, implicit differentiation of the relations $x^2 + y^2 = 9$ and $3xy^2 = x + y$                                                                                                           |  | SL5.6*  | SL5.6*<br>AL5.14  |  | v* AHL5.9 | *not implicit, just chain rule |
| Specialist | 3 and 4 | techniques of anti-differentiation and for the evaluation of definite integrals:<br>–anti-differentiation of $1/ x $ to obtain $\log x$                                                                                                                                                  |  | SL5.5   | SL5.5             |  | v SL5.5   |                                |
|            |         | –anti-differentiation by recognition that they are derivatives of corresponding inverse circular functions                                                                                                                                                                               |  |         |                   |  |           |                                |
| Specialist | 3 and 4 | –use of the substitution $u = g(x)$ to anti-differentiate expressions                                                                                                                                                                                                                    |  | SL5.10  | SL5.10<br>AL5.16  |  | v AHL5.11 |                                |
| Specialist | 3 and 4 | –use of the trigonometric identities $\sin(2ax) = 1(1 - \cos(2ax))$ , $\cos(2ax) = 1(1 + \cos(2ax))$ , in anti-differentiation techniques                                                                                                                                                |  | v       | v                 |  |           |                                |
|            |         | –anti-differentiation using partial fractions of rational functions                                                                                                                                                                                                                      |  | SL5.10  | SL5.10<br>AHL5.15 |  |           |                                |
| Specialist | 3 and 4 | application of integration, arc lengths of curves, areas of regions bounded by curves and volumes of solids of revolution of a region about either coordinate axis.                                                                                                                      |  | SL5.11* | SL5.11<br>AL5.17  |  | v AHL5.12 | *no volumes of revolutions     |
| Specialist | 3 and 4 | formulation of differential equations from contexts in, for example, physics, chemistry, biology and economics, in situations where rates are involved (including some differential equations whose analytic solutions are not required, but can be solved numerically using technology) |  | SL5.9*  | SL5.9*            |  | v AHL5.13 | *kinematics                    |
| Specialist | 3 and 4 | verification of solutions of differential equations and their representation using direction (slope) fields                                                                                                                                                                              |  |         |                   |  | v AHL5.15 |                                |
| Specialist | 3 and 4 | solution of simple differential equations of the form $dy/dx = f(x)$ , $dy/dx = g(y)$ , and in general differential equations of the form $dy/dx = f(x)g(y)$ using separation of variables and differential equations of the form $d^2y/dx^2 = f(x)$                                     |  |         | AHL5.18           |  |           |                                |

[illegible]

|                            |         |                                                                                                                                                                                                                                                                                                                                                                               |  |  |          |  |               |                                                                                   |
|----------------------------|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|----------|--|---------------|-----------------------------------------------------------------------------------|
| Specialist                 | 3 and 4 | inertial mass, momentum, including change of momentum (conservation of momentum and impulse are not required), force, resultant force, weight, action and reaction                                                                                                                                                                                                            |  |  |          |  |               |                                                                                   |
| Specialist                 | 3 and 4 | equations of motion using absolute units (Equations of motion should be described from a diagram, showing all the forces acting on the body, and then writing down the equation of motion. Extensions could include cases involving a system of two or more connected particles. Examples are to be restricted to rectilinear motion, including motion on an inclined plane.) |  |  |          |  |               |                                                                                   |
| Specialist                 | 3 and 4 | motion of a body, regarded as a particle under the action of concurrent coplanar forces (the case of equilibrium should be regarded as an application, where net force is zero).                                                                                                                                                                                              |  |  |          |  |               |                                                                                   |
| Probability and Statistics |         |                                                                                                                                                                                                                                                                                                                                                                               |  |  |          |  |               |                                                                                   |
| Specialist                 | 3 and 4 | for random variables X and Y, $E(aX + b) = aE(X) + b$ and $E(aX + bY) = aE(X) + bE(Y)$                                                                                                                                                                                                                                                                                        |  |  | AHL4.14* |  | v*<br>AHL4.14 | * not $E(aX + bY) = aE(X) + bE(Y)$                                                |
| Specialist                 | 3 and 4 | for random variables X and Y, $Var(aX + b) = a^2Var(X)$ and for independent random variables X and Y, $Var(aX + bY) = a^2Var(X) + b^2Var(Y)$                                                                                                                                                                                                                                  |  |  | AHL4.14* |  | v*<br>AHL4.14 | *for independent random variables X and Y, $Var(aX + bY) = a^2Var(X) + b^2Var(Y)$ |
| Specialist                 | 3 and 4 | for independent random variables X and Y with normal distributions then $aX + bY$ also has a normal distribution                                                                                                                                                                                                                                                              |  |  |          |  | v*<br>AHL4.15 | * AHL 4.15 n independent normal random variables                                  |
| Specialist                 | 3 and 4 | Concept of the sample mean $\bar{X}$ as a random variable whose value varies between samples where X is a random variable with mean $\mu$ and standard deviation $\sigma$                                                                                                                                                                                                     |  |  |          |  | v AHL4.15     |                                                                                   |



|            |         |                                                                                                                                                                                                                                                                                                                                                                                         |  |  |  |  |           |  |
|------------|---------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|--|-----------|--|
| Specialist | 3 and 4 | simulation of repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate properties of the distribution of $X$ across samples of a fixed size $n$ including its mean $\mu$ and its standard deviation $\sigma/\sqrt{n}$ (where $\mu$ and $\sigma$ are the mean and standard deviation of $X$ ) and its approximate normality if $n$ is large. |  |  |  |  | V AHL4.15 |  |
| Specialist | 3 and 4 | determination of confidence intervals for means and the use of simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain $\mu$                                                                                                                                                                         |  |  |  |  | V AHL4.16 |  |
| Specialist | 3 and 4 | construction of an approximate confidence interval where $s$ is the sample standard deviation and $z$ is the appropriate quantile for the standard normal distribution, in particular the 95% confidence interval as an example of such an interval where $z \approx 1.96$ (the term standard error may be used but is not required).                                                   |  |  |  |  | V AHL4.16 |  |
| Specialist | 3 and 4 | p values for hypothesis testing related to the mean                                                                                                                                                                                                                                                                                                                                     |  |  |  |  | V AHL4.18 |  |
| Specialist | 3 and 4 | Formulation of a null hypothesis and an alternative hypothesis                                                                                                                                                                                                                                                                                                                          |  |  |  |  | V AHL4.18 |  |
| Specialist | 3 and 4 | errors in hypothesis testing.                                                                                                                                                                                                                                                                                                                                                           |  |  |  |  | V AHL4.18 |  |

## **IB Standards not in the VCE**

Below are the standards that are not covered in each of the VCE courses.

### **VCE General Mathematics**

| Analysis SL                                 | Analysis HL                                 | Apps SL                            | Apps HL                          |
|---------------------------------------------|---------------------------------------------|------------------------------------|----------------------------------|
| SL1.1                                       | SL1.1                                       | SL1.1                              | SL1.1                            |
| SL 1.2*sum and notation                     | SL 1.2*sum and notation                     | SL 1.2*sum and notation            | SL 1.2*sum and notation          |
| SL1.3*sum and notation                      | SL1.3*sum and notation                      | SL1.3*sum and notation             | SL1.3*sum and notation           |
| SL1.5                                       | SL1.5                                       | SL1.5                              | SL1.5                            |
| SL1.6                                       | SL1.6                                       | SL1.6 Bounds, percentage error     | SL1.6 Bounds, percentage error   |
| SL1.7                                       | SL1.7                                       | SL1.8                              | SL1.8                            |
| SL1.8                                       | SL1.8                                       | SL2.1 Parallel and perpendicular   | AHL1.9                           |
| SL1.9                                       | SL1.9                                       | SL2.2                              | AHL1.10                          |
| SL2.1* Parallel, perpendicular, rearranging | AHL1.10                                     | SL2.3                              | AHL1.11                          |
| SL2.2                                       | AHL1.11                                     | SL2.4                              | AHL1.12                          |
| SL2.3                                       | AHL1.12                                     | SL2.5                              | AHL1.13                          |
| SL2.4                                       | AHL1.13                                     | SL2.6                              | AHL1.15                          |
| SL2.5                                       | AHL1.14                                     | SL3.1                              | SL2.1 Parallel and perpendicular |
| SL2.6                                       | AHL1.15                                     | SL3.5                              | SL2.2                            |
| SL2.7                                       | AHL1.16                                     | SL3.6                              | SL2.3                            |
| SL2.8                                       | SL2.1* Parallel, perpendicular, rearranging | SL4.1 sampling techniques and bias | SL2.4                            |
| SL2.9                                       | SL2.2                                       | SL4.3* constant change             | SL2.5                            |
| SL2.10                                      | SL2.3                                       | SL4.5                              | SL2.6                            |
| SL2.11                                      | SL2.4                                       | SL4.6                              | AHL2.7                           |
| SL3.1                                       | SL2.5                                       | SL4.7                              | AHL2.8                           |

|                       |                       |        |                                                 |
|-----------------------|-----------------------|--------|-------------------------------------------------|
| SL3.4*radians         | SL2.6                 | SL4.8  | AHL2.9                                          |
| SL3.5                 | SL2.7                 | SL4.9  | SL3.1                                           |
| SL3.6                 | SL2.8                 | SL4.10 | SL3.4 Radians                                   |
| SL3.7                 | SL2.9                 | SL4.11 | SL3.5                                           |
| SL3.8                 | SL2.10                | SL5.1  | SL3.6                                           |
| SL4.3*Constant change | SL2.11                | SL5.2  | AHL3.7                                          |
| SL4.5                 | AHL2.12               | SL5.3  | AHL3.8                                          |
| SL4.6                 | AHL2.12               | SL5.4  | AHL3.9                                          |
| SL4.7                 | AHL2.13               | SL5.5  | AHL3.10                                         |
| SL4.8                 | AHL2.14               | SL5.6  | AHL3.11                                         |
| SL4.9*                | AHL2.15               | SL5.7  | AHL3.12                                         |
| SL4.10                | AHL2.16               | SL5.8  | AHL3.13                                         |
| SL4.11                | SL3.1                 |        | AHL3.16 Chinese postman and travelling salesman |
| SL5.1                 | SL3.4*radians         |        | SL4.1 sampling techniques and bias              |
| SL5.2                 | SL3.5                 |        | AL4.3 * constant change                         |
| SL5.3                 | SL3.6                 |        | SL4.5                                           |
| SL5.4                 | SL3.7                 |        | SL4.6                                           |
| SL5.5                 | SL3.8                 |        | SL4.7                                           |
| SL5.6                 | AHL3.9                |        | SL4.8                                           |
| SL5.7                 | AHL3.10               |        | SL4.9                                           |
| SL5.8                 | AHL3.11               |        | SL4.10                                          |
| SL5.9                 | AHL3.12               |        | SL4.11                                          |
| SL5.10                | AHL3.13               |        | AHL4.12                                         |
| SL5.11                | AHL3.14               |        | AHL4.13                                         |
|                       | AHL3.15               |        | AHL4.14                                         |
|                       | AHL3.16               |        | AHL4.15                                         |
|                       | AHL3.17               |        | AHL4.16                                         |
|                       | AHL3.18               |        | AHL4.17                                         |
|                       | SL4.3*Constant change |        | AHL4.18                                         |
|                       | SL4.5                 |        | SL5.1                                           |
|                       | SL4.6                 |        | SL5.2                                           |

|  |         |  |         |
|--|---------|--|---------|
|  | SL4.7   |  | SL5.3   |
|  | SL4.8   |  | SL5.4   |
|  | SL4.9*  |  | SL5.5   |
|  | SL4.10  |  | SL5.6   |
|  | SL4.11  |  | SL5.7   |
|  | AHL4.13 |  | SL5.8   |
|  | AHL4.14 |  | AHL5.9  |
|  | SL5.1   |  | AHL5.10 |
|  | SL5.2   |  | AHL5.11 |
|  | SL5.3   |  | AHL5.12 |
|  | SL5.4   |  | AHL5.13 |
|  | SL5.5   |  | AHL5.14 |
|  | SL5.6   |  | AHL5.15 |
|  | SL5.7   |  | AHL5.16 |
|  | SL5.8   |  | AHL5.17 |
|  | SL5.9   |  | AHL5.18 |
|  | SL5.10  |  |         |
|  | SL5.11  |  |         |
|  | AHL5.12 |  |         |
|  | AHL5.13 |  |         |
|  | AHL5.14 |  |         |
|  | AHL5.15 |  |         |
|  | AHL5.16 |  |         |
|  | AHL5.17 |  |         |
|  | AHL5.18 |  |         |
|  | AHL5.19 |  |         |

# VCE Specialist Mathematics

| Analysis SL | Analysis HL | Apps SL | Apps HL |
|-------------|-------------|---------|---------|
| SL1.1       | SL1.1       | SL1.1   | SL1.1   |
| SL1.5       | SL1.5       | SL1.5   | SL1.5   |
| SL1.7       | SL1.7       | SL1.6   | SL1.6   |
| SL2.1       | AHL1.11     | SL1.7   | SL1.7   |
| SL2.2       | AHL1.16     | SL1.8   | SL1.8   |
| SL2.3       | SL2.1       | SL2.1   | AHL1.9  |
| SL2.4       | SL2.2       | SL2.2   | AHL1.10 |
| SL2.5       | SL2.3       | SL2.3   | AHL1.15 |
| SL2.6       | SL2.4       | SL2.4   | SL2.1   |
| SL2.7       | SL2.5       | SL2.5   | SL2.2   |
| SL2.9       | SL2.6       | SL2.6   | SL2.3   |
| SL2.10      | SL2.7       | SL3.1   | SL2.4   |
| SL3.1       | SL2.9       | SL3.4   | SL2.5   |
| SL3.4       | SL2.10      | SL3.5   | SL2.6   |
| SL3.5       | AHL2.12     | SL3.6   | AHL2.7  |
| SL3.6       | AHL2.14     | SL4.2   | AHL2.9  |
| SL3.8       | AHL2.15     | SL4.3   | AHL2.10 |
| SL4.2       | SL3.1       | SL4.4   | SL3.1   |
| SL4.3       | SL3.4       | SL4.5   | SL3.4   |
| SL4.5       | SL3.5       | SL4.6   | SL3.5   |
| SL4.6       | SL3.6       | SL4.9   | SL3.6   |
| SL4.8       | SL3.8       | SL4.10  | AHL3.7  |
| SL4.9*      | AHL3.11     | SL4.11  | AHL3.8  |
| SL4.10      | AHL3.15     | SL5.1   | AHL3.16 |
| SL4.11      | AHL3.16     | SL5.2   | SL4.2   |
| SL5.1       | AHL3.17     | SL5.3   | SL4.3   |
| SL5.2       | AHL3.18     | SL5.4   | SL4.4   |
| SL5.3       | SL4.2       | SL5.6   | SL4.5   |
| SL5.4       | SL4.3       | SL5.7   | SL4.6   |
| SL5.8       | SL4.5       | SL5.8   | SL4.9   |
| SL5.10      | SL4.6       |         | SL4.10  |

|  |                |  |         |
|--|----------------|--|---------|
|  | SL4.8          |  | SL4.11  |
|  | SL4.9          |  | AHL4.12 |
|  | SL4.10         |  | AHL4.13 |
|  | SL4.11         |  | AHL4.17 |
|  | <i>AHL4.13</i> |  | AHL4.19 |
|  | SL5.1          |  | SL5.1   |
|  | SL5.2          |  | SL5.2   |
|  | SL5.3          |  | SL5.3   |
|  | SL5.4          |  | SL5.4   |
|  | SL5.8          |  | SL5.6   |
|  | SL5.10         |  | SL5.7   |
|  | AHL5.12        |  | SL5.8   |
|  | AHL5.13        |  | AHL5.14 |
|  | AHL5.16        |  | AHL5.17 |
|  | AHL5.19        |  | AHL5.18 |