Comparison of South Australia Mathematics and the new IB Mathematics Courses

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IBDP MATHEMATICS: APPLICATIONS AND INTERPRETATION SYLLABUS

Topic 1 - Number and Algebra	SL	1.1	11 General		
	SL	1.2	11 Mathematics		
	SL	1.3	11 Mathematics		
	SL	1.4	11 General	11 Mathemati	12 General
	SL	1.5	11 Mathematics		
	SL	1.6	11 General		
	SL	1.7	12 General		
	SL	1.8	11 Mathemati	12 Specialist	
	AHL	1.9	11 Mathematics		
	AHL	1.10	11 Mathematics		
	AHL	1.11	11 Mathematics		
	AHL	1.12	11 Mathematics		
	AHL	1.13	12 Specialist	11 Mathematics	
	AHL	1.14	11 Mathemati	11 General	
	AHL	1.15	Not covered		
Topic 2 - Functions	SL	2.1	11 General	11 Mathematics	
	SL	2.2	11 Mathemati	12 Specialist	
	SL	2.3	11 Mathematics		
	SL	2.4	11 Mathematics		
	SL	2.5	11 Mathemati	11 General	
	SL	2.6	Not covered		
	AHL	2.7	12 Specialist		
	AHL	2.8	11 Mathematics		
	AHL	2.9	11 Mathemati	12 Methods	
	AHL	2.10	12 Methods	* only partially	
Topic 3 - Geometry and Trigonometry	SL	3.1	11 Mathemati	11 General	
	SL	3.2	11 Mathemati	11 General	
	SL	3.3	11 Mathemati	11 General	
	SL	3.4	11 Mathemati	11 General	
	SL	3.5	11 Mathematics		
	SL	3.6	Not covered		
	AHL	3.7	11 Mathematics		
	AHL	3.8	11 Mathematics		
	AHL	3.9	11 Mathematics		
	AHL	3.10	11 Mathemati	* 2-dimensional	
	AHL	3.11	12 Specialist		
	AHL	3.12	12 Specialist		
	AHL	3.13	11 Mathemati	12 Specialist	
	AHL	3.14	11 General	* only partially	
	AHL	3.15	11 General	* only partially	
	AHL	3.16	11 General	* only partially	
4 - Statistics and Probability	SL	4.1	11 General	11 Mathematics	
	SL	4.2	11 General		
	SL	4.3	11 General	* only partially	
	SL	4.4	12 General		
	SL	4.5	Year 10-10A	11 Mathematics	
	SL	4.6	Year 10-10A	11 Mathematics	
	SL	4.7	12 Methods	11 Mathematics	
	SL	4.8	12 Methods		
	SL	4.9	12 Methods	12 General	
	SL	4.10	Not covered		
	SL	4.11	Not covered		
	AHL	4.12	Not covered		
	AHL	4.13	Not covered		

Topic	AHL	4.14	Not covered	
	AHL	4.15	12 Methods	
	AHL	4.16	12 Methods	
	AHL	4.17	Not covered	
	AHL	4.18	Not covered	
	AHL	4.19	12 General	* only partially
Topic 5 - Calculus	SL	5.1	11 Mathemati	12 Methods
	SL	5.2	11 Mathemati	12 Methods
	SL	5.3	11 Mathemati	12 Methods
	SL	5.4	11 Mathemati	12 Methods
	SL	5.5	12 Methods	
	SL	5.6	11 Mathemati	12 Methods
	SL	5.7	11 Mathemati	12 Methods
	SL	5.8	12 Methods	
	AHL	5.9	12 Methods	
	AHL	5.10	12 Methods	
	AHL	5.11	12 Methods	12 Specialist
	AHL	5.12	12 Methods	12 Specialist
	AHL	5.13	12 Methods	11 Mathematics
	AHL	5.14	12 Specialist	
	AHL	5.15	12 Specialist	
	AHL	5.16	Not covered	
	AHL	5.17	Not covered	
	AHL	5.18	Not covered	

IBDP MATHEMATICS: ANALYSIS AND APPROACHES SYLLABUS

Topic 1 - Number and Algebra	SL	1.1	11 General		
	SL	1.2	11 Mathematics		
	SL	1.3	11 Mathematics		
	SL	1.4	11 General	11 Mathematics	12 General
	SL	1.5	11 Mathematics		
	SL	1.6	11 Mathemati	12 Specialist	
	SL	1.7	11 Mathemati	* Not change of base	
	SL	1.8	11 Mathematics		
	SL	1.9	11 Mathematics		
	AHL	1.10	11 Mathemati	* only partially	
	AHL	1.11	Not covered		
	AHL	1.12	11 Mathematics		
	AHL	1.13	12 Specialist	11 Mathematics	* Not Euler's form
	AHL	1.14	11 Mathemati	12 Specialist	
	AHL	1.15	12 Specialist	11 Mathematics	
	AHL	1.16	12 Specialist		
Topic 2 - Functions	SL	2.1	11 General	11 Mathematics	
	SL	2.2	11 Mathemati	12 Specialist	
	SL	2.3	11 Mathematics		
	SL	2.4	11 Mathematics		
	SL	2.5	12 Methods	12 Specialist	
	SL	2.6	11 Mathematics		
	SL	2.7	11 Mathematics		
	SL	2.8	11 Mathemati	12 Specialist	
	SL	2.9	11 Mathemati	12 Methods	
	SL	2.10	11 Mathematics		
	SL	2.11	11 Mathematics		
	AHL	2.12	12 Specialist	11 Mathematics	
	AHL	2.13	12 Specialist		
	AHL	2.14	12 Specialist	* only partially	
	AHL	2.15	Not covered		
	AHL	2.16	12 Specialist	* only partially	
Topic 3 - Geometry and Trigonometry	SL	3.1	11 Mathemati	11 General	
	SL	3.2	11 Mathemati	11 General	
	SL	3.3	11 Mathemati	11 General	
	SL	3.4	11 Mathemati	11 General	
	SL	3.5	11 Mathematics		
	SL	3.6	11 Mathematics		
	SL	3.7	11 Mathematics		
	SL	3.8	11 Mathematics		
	AHL	3.9	11 Mathemati	12 Specialist	
	AHL	3.10	11 Mathematics		
	AHL	3.11	11 Mathematics		
	AHL	3.12	12 Specialist	11 Methods (2D)	
	AHL	3.13	11 Mathemati	12 Specialist	
	AHL	3.14	12 Specialist		
	AHL	3.15	12 Specialist		
	AHL	3.16	12 Specialist		
	AHL	3.17	12 Specialist	* only partially	
	AHL	3.18	12 Specialist		
bability	SL	4.1	11 General	11 Mathematics	
	SL	4.2	11 General		
	SL	4.3	11 Mathemati	* only partially	
	SL	4.4	12 General		

Topic 4 - Statistics and Pro	SL	4.5	Year 10-10A	11 Mathematics	
	SL	4.6	Year 10-10A	11 Mathematics	
	SL	4.7	12 Methods	11 Mathematics	
	SL	4.8	12 Methods		
	SL	4.9	12 Methods	12 General	
	SL	4.10	12 General		
	SL	4.11	Not covered		
	SL	4.12	12 Methods		
	AHL	4.13	Not covered		
	AHL	4.14	12 Methods	* only partially	
Topic 5 - Calculus	SL	5.1	11 Mathemati	12 Methods	
	SL	5.2	11 Mathemati	12 Methods	
	SL	5.3	11 Mathemati	12 Methods	
	SL	5.4	11 Mathemati	12 Methods	
	SL	5.5	12 Methods		
	SL	5.6	12 Methods		
	SL	5.7	12 Methods		
	SL	5.8	12 Methods	11 Mathematics	
	SL	5.9	12 Methods	11 Mathematics	
	SL	5.10	12 Methods	* only partially	
	SL	5.11	12 Methods	12 Specialist	
	AHL	5.12	12 Methods	11 Mathematics	* only partially
	AHL	5.13	Not covered		
	AHL	5.14	12 Specialist		
	AHL	5.15	12 Specialist	* only partially	
	AHL	5.16	12 Specialist	* only partially	
	AHL	5.17	12 Methods	12 Specialist	
	AHL	5.18	12 Specialist	* only partially	
	AHL	5.19	Not covered		

Stage 1 General Mathematics	Unit (Topic)	Content	Included in Prior learning	Included in Analysis SL	Included in Analysis HL	Included in Apps SL	Included in Apps HL	Comments
Topic 1 Investing and Borrowing	**Subtopic 1.1 Investing for Interest**	Why invest money in financial institutions? Where can money be invested? • Discussion of financial institutions • Fees and charges • Types of investment						Not specifically mentioned in SL 1.4
		How is simple interest calculated, and in which situations is it used? • Using the simple interest formula to find the ◆ simple interest ◆ principal ◆ interest rate ◆ time invested in years ◆ total return		✓	✓	✓	✓	SL 1.4 common content
		How does compound interest work?		✓	✓	✓	✓	SL 1.4 common content
		How is compound interest calculated? • Derivation of the compound interest formula • Using the formula to find future value, interest earned, and present value		✓	✓	✓	✓	SL 1.4 common content
		• Effects of changing the compounding period • Annualised rates for comparison of investments		✓*	✓	✓*	✓	Annualised rates are not specifically mentioned in SL 1.4
		• Using electronic technology to find the ◆ future value ◆ present value ◆ interest rate ◆ time ◆ comparison rates on savings		✓	✓	✓	✓	SL 1.4 common content

		Which is the better option: simple interest or compound interest?		✓	✓	✓	✓	SL 1.4 common content
	Subtopic 1.2: Investing in shares	How can the share market be used to make money from the money someone already has? • Share market information • Costs and risks • Buying and selling shares • Break-even price • Using a brokerage rate $BE=\frac{b(1+1.1r)}{1-1.1r}$ • Using a flat fee for brokerage $BE=\left(\frac{2.2f}{n}\right)+b$ • Calculation of the dividend return from shares given the percentage dividend or the dividend per share						**Shares** not mentioned in the IB Guide
	Subtopic 1.3: Return on investment	• Expressing the return on an investment as a percentage of the original investment						**Shares** not mentioned in the IB Guide
		• The effect of tax and inflation on real growth of an investment		✓*	✓	✓*	✓	Inflation mentioned in SL 1.4, but not tax
	Subtopic 1.4: Costs of borrowing	Why do many people use credit to buy items rather than saving for them? What types of credit are available? What is the total cost of using credit?						Loans only mentioned in "**Other contexts**" below SL 1.4 in the guide.

		How much does a personal loan cost? • Extra fees and charges • Administration fees • Interest When is it better to borrow than save?						Loans only mentioned in "**Other contexts**" below SL 1.4 in the guide.
Topic 2: Measurement	**Subtopic 2.1: Application of measuring devices and units of measurement**	Application of common measuring devices, the metric system, its units and conversion between them.	✓					prior knowledge?
		How should accuracy be considered in measurement? • Estimation and approximation • Rounding off to a given number of significant figures • Calculation of absolute and percentage errors using error tolerances				✓	✓	AISL 1.6
		How are very large and very small values in measurement expressed? • Scientific notation		✓	✓	✓	✓	SL 1.1 common content
	Subtopic 2.2: Perimeter and area of plane shapes	How can we use Pythagoras' theorem to solve problems involving right-angled triangles?		✓	✓	✓	✓	SL 3.3 common content
		How can knowing the perimeter and area of a two-dimensional shape help with solving a problem? • Calculating circumferences and perimeters of standard and composite shapes (including circles, sectors, quadrilaterals, and triangles) • Calculating areas of standard and composite shapes (including circles, sectors, quadrilaterals, ovals, trapeziums, and triangles) • Converting between units of measurement for area	✓	✓	✓	✓	✓	prior knowledge? And SL 3.4 common content

		How can the area of an irregular plane shape be estimated? • Approximation using a simple mathematical shape (circle, oval, rectangle, triangle, etc.)						Not specifically mentioned.
		• Simpson's rule $A = \frac{1}{3}w\left(d_1 + 4d_2 + d_3\right)$						Not specifically mentioned.
	Subtopic 2.3: Volume and surface area of solids	How is the amount of space an object occupies or the amount of liquid a container will hold determined? • Calculating volume or capacity for standard and composite solids including prisms, cylinders, cones, pyramids, and spheres.		✓	✓	✓	✓	SL 3.1
		• Converting between units for volume and capacity						Prior knowledge?
		• Estimating the volume of an irregular solid using an appropriate mathematical model						not specifically mentioned
		How is the area of the outside surface of a solid shape determined? • Calculating surface area for standard and composite solids including prisms, cylinders, cones, pyramids, and spheres		✓	✓	✓	✓	SL 3.1 common content
	Subtopic 2.4: Scale and rates	How does a scale factor work? • Using a scale factor to calculate actual and scaled measurements • Drawing scaled diagrams • Determining the scale factor needed or used						Prior knowledge?
		• Scaling areas and volumes						Prior knowledge?

		What is a rate? What does it measure? • Rates of change with time, particularly speed and flow rates • Other rates, particularly density		✓*	✓	✓*	✓	SL 5.1 common content "rates of change" density not mentioned explicitly.
		• Converting between units for a rate						?assume this is included in IB
Topic 3: Statistical investigation	**Subtopic 3.1: The statistical investigation process**	What are some examples of situations in which statistics are used to analyse and investigate problems? The statistical process: • identifying the problem • formulating the method of investigation • collecting data • analysing the data		✓	✓	✓	✓	SL 4.1 common content
		• interpreting the results and forming a conjecture • considering the underlying assumptions						?assume this is included in IB
	Subtopic 3.2: Sampling and collecting data	What is a sample and what is the purpose of sampling?		✓	✓	✓	✓	SL 4.1 common content
		What is bias and how can it occur in sampling?		✓	✓	✓	✓	SL 4.1 common content
		What methods of sampling are there? • Simple random, stratified, and systematic sampling methods		✓	✓	✓	✓	SL 4.1 common content
	Subtopic 3.3: Classifying and organising data	Categorical data • Ordinal • Nominal						not specifically mentioned

		Numerical data • Discrete • Continuous How can data of the different types be appropriately organised and displayed? • Categorical data — tables and bar or pie charts • Numerical data — dot plot, stem plot, histogram		✓	✓	✓	✓	SL 4.2 common content
		What is an outlier? How should outliers be dealt with?		✓	✓	✓	✓	SL 4.1 common content
	Subtopic 3.4: The shape, location, and spread of distributions of numerical data	What does the distribution of data within a data set look like?						?assume this is included in IB
		What is meant by 'average'? • Measures of central location (median and mean)		✓	✓	✓	✓	SL 4.3 common content
		How do you decide on the most appropriate measure of 'average'? When can these measures become unreliable or misleading?		✓	✓	✓	✓	SL 4.3 common content
		Do sets of data with the same 'average' necessarily tell the same story? • Box-and-whisker plots		✓	✓	✓	✓	SL 4.1 - 4.4 common content
		• Measures of spread (range, interquartile range, standard deviation) • Outliers		✓	✓	✓	✓	

		What influence does sample size have on the reliability of findings? • Sample statistics compared with population parameters						?assume this is included in IB
	Subtopic 3.5: Forming and supporting conjectures across two or more groups	How do the statistical techniques and measures learnt help to argue whether a claim is true or false? Analysis of numerical data: • graphical representation • dealing with outliers • shape of the distribution(s) • measures of centre and spread • argument to support the conjecture Analysis of categorical data: • table of counts • graphical representation • identification of the mode • calculation of proportions • argument to support the conjecture		✓*	✓	✓*	✓	Categorical data is not specifically mentioned
Topic 4: Applications of trigonometry	**Subtopic 4.1: Similarity**	In what kinds of problems are triangles important?	✓					
		How many measurements are required to determine a triangle uniquely?	✓					
		Under what conditions can two triangles be proved to be similar?	✓					
		How can similarity be used to solve problems?	✓					

	Subtopic 4.2: Right triangle geometry	What mathematical tools are there for solving problems involving right-angled triangles? • Pythagoras' theorem • Trigonometric ratios		✓	✓	✓	✓	SL 3.2 common content
	Subtopic 4.3: Area of triangles	How is the area of a non-right triangle found if the perpendicular to a side cannot be measured easily or accurately? $Area = \frac{1}{2} ab \sin C$		✓	✓	✓	✓	SL 3.2 common content
		How can the area of a triangle be determined from its three sides? • Heron's rule $Area = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$						not specifically mentioned
	Subtopic 4.4: Solving problems with non-right triangles	How are problems solved in which the triangles involved are not right-angled?		✓	✓	✓	✓	SL 3.2 common content
		The cosine rule • Solving for the third side when two sides and the included angle are known $a^2 = b^2 + c^2 - 2bc \cos A$		✓	✓	✓	✓	SL 3.2 common content
		• Solving for angles when the three sides are known		✓	✓	✓	✓	SL 3.2 common content

		The sine rule • Solving triangles where two sides and the non-included angle are known $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$		✓	✓	✓	✓	SL 3.2 common content
		• Solving triangles where two angles and one side are known		✓	✓	✓	✓	SL 3.2 common content
		• Solving problems involving direction and bearings	✓					
Topic 5: Linear and exponential functions and their graphs	**Subtopic 5.1: Linear functions and graphs**	What is the nature of a linear relationship? • Successive addition or subtraction of a constant value		✓	✓	✓	✓	SL 2.1 common content
		How can problems that involve linear functions be represented? • Contextual description		✓	✓	✓	✓	SL 2.1 common content AISL2.5 also "linear models"
		In what other ways can such a problem be represented? • Numerical table of values		✓	✓	✓	✓	SL 2.1 common content
		• Graphical representation ◆ Slope and intercepts in context ◆ Determining x or y value from a linear graph, given the other corresponding value		✓	✓	✓	✓	SL 2.1 common content

		• Algebraic formula ◆ Developing a linear formula from a word description ◆ Substitution and evaluation ◆ Rearrangement of linear equations ◆ Solving linear equations		✓	✓	✓	✓	SL 2.1 common content
		What are the links between the four ways of representing a linear relationship?	✓					
	Subtopic 5.2: Exponential functions and graphs	What is geometric growth or decay? • Successive multiplication by a constant positive value • Powers		✓*	✓	✓	✓	AI SL 2.5, AASL 2.9
		How does this kind of growth or decay differ from that seen in linear relationships?		✓*	✓	✓	✓	AI SL 2.5, AASL 2.9
		What are the different representations for an exponential function and how do we move between them? • Features of the graph • The algebraic formula $y = a.b^x$		✓	✓	✓	✓	AI SL 2.5, AASL 2.9
		How can the model be used to solve problems in context? • Compound interest		✓	✓	✓	✓	SL 1.4 common content and further technology use in AISL 1.7 also
		• Other growth contexts (population growth, inflation, etc.) • Decay contexts (radioactive decay depreciation, cooling etc.)		✓	✓	✓	✓*	SL 2.5 common content AIHL 2.9 "half-life"
		• Finding percentage growth or decay						included??

Topic 6: Matrices and networks	**Subtopic 6.1: Matrix arithmetic and costing applications**	What is a matrix?					✓	AIHL 1.14
		How is information organised in a matrix? • Columns and rows in a matrix • Order (or dimensions) of a matrix In what ways can costing and stock information in matrix form be manipulated?					✓	AIHL 1.14
		• Adding and subtracting matrices • Multiplication by a scalar					✓	AIHL 1.14
		• Matrix multiplication ◆ using a row or column matrix					✓	AIHL 1.14
		◆ using matrices of higher order					✓	AIHL 1.14
		◆ multiplying by a row or column matrix of 1s					✓	AIHL 1.14
		• Using electronic technology to do matrix arithmetic					✓	AIHL 1.14
		How can matrices be used to solve problems in costing and inventory control?						?assume this is in IB
		How useful is the matrix model?						?assume this is in IB
	Subtopic 6.2: Networks	What are networks?					✓	AIHL 3.14
		What information is given in a network diagram? • Reading information from a network diagram					✓	AIHL 3.14
		• Deducing relationships					✓	AIHL 3.14
		• Using appropriate terminology					✓	AIHL 3.14

		How can networks be used to represent situations in which there is a problem to be solved? • Connectivity networks • Flow networks					✓	AIHL 3.14
		How many paths are there through a directed network? • With and without restrictions					✓	AIHL 3.15
		What is the shortest or longest path through a network? • With and without restrictions					✓	AIHL 3.15
		What is the cheapest way to connect up a set of points if there is more than one option available? • Spanning trees – using 'greedy' and Prim's algorithms to find the minimum spanning tree in a connectivity network					✓	AIHL 3.16
		What is the maximum flow that can be achieved through a network of conduits? • Use of an algorithm to find maximum flow				✓?	✓	?possibly a low-level example of network problems in AIHL ??possibly related to low-level Voronoi diagram problems
Topic 7: Open topic		Schools may choose to develop a topic that is relevant to their local context.						??
✓*	partially included							
✓	included							

Stage 2 General Mathematics	Unit (Topic)	Content	Included in Prior learning	Included in Analysis SL	Included in Analysis HL	Included in Apps SL	Included in Apps HL	Comments
Topic 1: Modelling with linear relationships	**Subtopic 1.1: Simultaneous linear equations**	Consider the different ways a linear function can be represented and the links between them: • Contextual description • Numerical sequence • Graph • Algebraic formula For a problem with two independent variables, consider how much information is required to determine a unique solution.	✓					
		Consider how contextual problems involving simultaneous linear equations can be solved efficiently. Using electronic technology for: graphing, using the equation solver functionality.	✓					
		Non-unique solutions	✓					
	Subtopic 1.2: Linear programming	How can linear functions be used to optimise a situation where we have control of two variables? Setting up the constraints (with inequalities) and an objective function. Graphing the feasible region. Finding the optimal solution.						No **linear programming** in IB
		Considering wastage						No **linear programming** in IB
		How do we deal with an optimal solution that is not achievable because only discrete values are allowed?						No **linear programming** in IB
		What happens to the optimal solution if the original parameters change?						No **linear programming** in IB
Topic 2: Modelling with matrices	**Subtopic 2.1: Application of matrices to network problems**	Consider how can a matrix be used to show the connections in a network.					✓	AIHL 3.14 Graph theory focus
		Connectivity matrices					✓	AIHL 3.15 Graph theory focus
		Consider how matrix operations help to find the number of indirect connections in a network.					✓	AIHL 3.15 Graph theory focus
		Powers of matrices and multi-stage connections					✓	AIHL 3.15 Graph theory focus

		Limitations of using higher powers							No specific **Dominance matrix** application mentioned in the IB guide.
		Consider of what use weighted sums of the powers of connectivity matrices are.							No specific **Dominance matrix** application mentioned in the IB guide.
		· Measures of efficiency or redundancy							No specific **Dominance matrix** application mentioned in the IB guide.
		Prediction in dominance relationships							No specific **Dominance matrix** application mentioned in the IB guide.
		Reasonableness of weightings and limitations of the model							No specific **Dominance matrix** application mentioned in the IB guide.
	Subtopic 2.2: Application of matrices to transition problems	Transition matrices and its properties.						✓	AIHL 4.19 (AIHL goes further, considering eigenvectors and eigenvalues)
		2 by 2 systems						✓	AIHL 4.19 (AIHL goes further, considering eigenvectors and eigenvalues)
		Consider how can future trends be predicted.						✓	AIHL 4.19 (AIHL goes further, considering eigenvectors and eigenvalues)
		Consider what happens in the long run in a transition model. The steady state.						✓	AIHL 4.19 (AIHL goes further, considering eigenvectors and eigenvalues)
		Consider the effect changes to the initial conditions have on the steady state.						✓	AIHL 4.19 (AIHL goes further, considering eigenvectors and eigenvalues)
		3 by 3 or higher order systems						✓	AIHL 4.19 (AIHL goes further, considering eigenvectors and eigenvalues)

		Consider the limitations of the transition matrix model.					✓	AIHL 4.19 (AIHL goes further, considering eigenvectors and eigenvalues)
Topic 3: Statistical models	**Subtopic 3.1: Bivariate statistics**	Consider how bivariate data be modelled when the relationship appears linear but is not perfect.		✓	✓	✓	✓	SL 4.4 common content
		· The statistical investigation process		✓	✓	✓	✓	SL 4.4 common content
		· Independent and dependent variables		✓	✓	✓	✓	SL 4.4 common content
		· Scatter plots		✓	✓	✓	✓	SL 4.4 common content
		Correlation coefficients		✓	✓	✓	✓	SL 4.4 common content
		The effects of outliers		✓	✓	✓	✓	SL 4.1 and SL 4.4 common content
		Causality		✓	✓	✓	✓	SL 4.4 common content
		Linear regression		✓	✓	✓	✓	SL 4.4 common content
		identification and interpretation of the slope and intercept of the graph of the linear equation in the context of the model		✓	✓	✓	✓	SL 4.4 common content
		Residual plots						**Not mentioned in the IB Guide**
		Exponential regression y=a*(b^x)					✓	AIHL 2.9 Exponential models and AIHL 4.13 (AIHL 4.13 goes further and includes quadratic, cubic, exponential, power and sine regression)
		interpretation of the values of '*a*' and '*b*'					✓	AIHL 4.13 (AIHL goes further and includes quadratic, cubic, exponential, power and sine regression)
		Interpolation and extrapolation, reliability, and interpretation of predicted results					✓	AIHL 4.13 (AIHL goes further and includes quadratic, cubic, exponential, power and sine regression)
	Subtopic 3.2: The normal distribution	Parameters μ (mean) and σ (standard deviation), Bell shape and symmetry about the mean		✓	✓	✓	✓	SL 4.9 common content

		Consider why so many observed sets of data appear normally distributed. Quantities that arise as the sum of a large number of independent random variables can be modelled as normal distributions.		✓	✓	✓	✓	SL 4.9 common content
		Consider why normal distributions are important. The variation in many quantities occurs in an approximately normal manner. Normal distributions may be used to make predictions and answer questions that relate to such quantities		✓	✓	✓	✓	SL 4.9 common content
		Consider how the characteristics of the normal distribution can be used for prediction. 68:95:99.7% rule.		✓	✓	✓	✓	SL 4.9 common content
		Calculation of area under the curve, looking at the position of one, two, and three standard deviations from the mean		✓	✓	✓	✓	SL 4.9 common content
		Calculation of non-standard proportions		✓	✓	✓	✓	SL 4.9 common content
		Calculation of values on the distribution, given the area under the curve		✓	✓	✓	✓	SL 4.9 common content
Topic 4: Financial models	**Subtopic 4.1: Models for saving**	The compound interest model is used to plan for the future. * On GDC using Financial Mode		✓	✓	✓	✓	SL 1.4 common content and further technology use in AISL 1.7 also
		· Finding *FV* , *PV* , *n*, and *I*		✓	✓	✓	✓	SL 1.4 common content and further technology use in AISL 1.7 also
		Consider how the compound interest model can be improved to make it more realistic and flexible.		✓	✓	✓	✓	SL 1.4 common content and further technology use in AISL 1.7 also
		Future valued annuities		✓	✓	✓	✓	SL 1.4 common content and further technology use in AISL 1.7 also
		Calculation of: Future value, The regular deposit, number of periods, interest rate, the value of the accumulated savings after a given period.		✓	✓	✓	✓	SL 1.4 common content and further technology use in AISL 1.7 also
		Total interest earned		✓	✓	✓	✓	SL 1.4 common content and further technology use in AISL 1.7 also

		Consider what factors should be considered when selecting an investment.		✓	✓	✓	✓	SL 1.4 common content and further technology use in AISL 1.7 also
		Interest as part of taxable income, including calculations						**Not specifically mentioned in SL 1.4**
		The effects of inflation, including calculations		✓	✓	✓	✓	SL 1.4 common content
		Institution and government charges						**Not specifically mentioned in SL 1.4**
		Comparison of two or more investments involving nominal and/or flat interest by conversion to an equivalent annualised rate (effective rate)						**Not specifically mentioned in SL 1.4**
		Consider how can a regular income be provided from savings? Annuities. Superannuation.						**Not specifically mentioned in SL 1.4**
	Subtopic 4.2: Models for borrowing	Consider if money must be borrowed, how much will it cost?		✓	✓	✓	✓	Loans only mentioned in "**Other contexts**" below SL 1.4 in the guide.
		Interest-only loans and sinking funds						Loans only mentioned in "**Other contexts**" below SL 1.4 in the guide.
		Reducing-balance loans, finding the repayment for a given loan, calculating total interest paid, the size of an outstanding debt after a given time.		✓	✓	✓	✓	Loans only mentioned in "**Other contexts**" below SL 1.4 in the guide.
		Consider how could the amount of interest paid on a loan can be reduced.		✓	✓	✓	✓	Loans only mentioned in "**Other contexts**" below SL 1.4 in the guide.
		Finding the effect of: increasing the frequency of payments, increasing the value of the payments, reducing the term of the loan, paying a lump sum off the principal owing, changing interest rates, using offset accounts		✓	✓	✓	✓	Loans only mentioned in "**Other contexts**" below SL 1.4 in the guide.
		Consider: Is the nominal rate of interest quoted by a bank what is really being paid on a loan?		✓	✓	✓	✓	Loans only mentioned in "**Other contexts**" below SL 1.4 in the guide.
		Loan interest rates, including variable rate, fixed rate, and others. Interest paid. Calculation of the comparison rates for two or more loans to determine the most appropriate option.		✓	✓	✓	✓	Loans only mentioned in "**Other contexts**" below SL 1.4 in the guide.

Topic 5: Discrete models	**Subtopic 5.1: Critical path analysis**	For a job requires the completion of a series of tasks with set precedence, what is the minimum time in which this job can be finished?						Although a precedence network is a graph, this particular application is possibly not covered in the IB Graph Theory content in AIHL 3.16
		Precedence tables. Drawing directed networks. Dummy links.						Although a precedence network is a graph, this particular application is possibly not covered in the IB Graph Theory content in AIHL 3.16
		Critical tasks. Forward and backward scans. Minimum completion time. Critical path.						Although a precedence network is a graph, this particular application is possibly not covered in the IB Graph Theory content in AIHL 3.16
		Earliest and latest starting times for individual tasks. Slack time.						Although a precedence network is a graph, this particular application is possibly not covered in the IB Graph Theory content in AIHL 3.16
	Subtopic 5.2: Assignment problems	Assignment problems deal with allocating tasks in a way that minimises ‘costs’ (note that ‘costs’ can be measurements such as time or distance, as well as money). For example, if the times in which four swimmers each do 50 metres of each of the four different strokes are known, how should they be placed in a medley relay to minimise the total time for them to complete the race?						**Not mentioned in the IB Guide.**
		The Hungarian algorithm for finding the optimum solution.						**Not mentioned in the IB Guide.**
		Finding minimum cost. Finding maximum profit. Non-square arrays.						**Not mentioned in the IB Guide.**
Topic 6: Open topic		*Schools may choose to develop a topic that is relevant to their own local context. When this option is undertaken, the open topic developed replaces Topic 2: Modelling with matrices.*						

		* Shares topic							**Not mentioned in the IB Guide.**
√*	partially included								
√	included								

Stage 1 Mathematics	Unit (topics)	Content	Included in Prior learning	Included in Analysis SL	Included in Analysis HL	Included in Apps SL	Included in Apps HL	Comments
Topic 1: Functions and graphs	**Subtopic 1.1: Lines and linear relationships**	• The equation of a straight line • from two points • from a slope and a point • parallel and perpendicular to a given line through some other point		✓	✓	✓	✓	SL 2.1 common content
		• Slope (m) as a rate of growth • y-intercept (c)		✓	✓	✓	✓	SL 2.1 common content
		• Solve simultaneous linear equations, graphically and algebraically • Find the points of intersection between two coincident straight lines		✓	✓	✓*	✓*	Intersections found using technology in SL 2.4 common content. AASL 2.10 - solving equations both graphically and analytically.
	Subtopic 1.2: Inverse proportion	Features of the graph of $y = \frac{1}{x}$		✓	✓	✓*	✓*	AASL 2.8. SL 2.4 common content mentions "determine key features of graphs....vertical and horizontal asymptotes using technology."
		Featuresof horizontal and vertical asymptotes includng translations.		✓	✓	✓*	✓*	AASL 2.8, AASL 2.11. SL 2.4 common content mentions "determine key features of graphs....vertical and horizontal asymptotes using technology."
	Subtopic 1.3: Relations	Equations of circles in both centre/radius and expanded form	✓					
	Subtopic 1.4: Functions	The concept of a function (and the concept of the graph of a function) • Domain and range • The use of function notation • Dependent and independent variables		✓	✓	✓	✓	SL 2.2 common content
		Recognise the distinction between functions and relations.						not explicitly mentioned but suspect included in SL 2.2.
Topic 2: Polynomials	**Subtopic 2.1: Quadratic relationships**	Quadratic relationships in everyday situations		✓	✓	✓	✓	AISL2.5, AASL2.6 (as examples).
		Features of the graph of y=x^2 and the relationships between y = a(x-b)^2+c and y = (x-a)(x-b).		✓	✓	✓*	✓*	AISL2.5(technology use focus for finding roots), AASL2.6.

		• Factorisation of quadratics of the form ax^2+bx+c and hence determine zeros • The quadratic formula to determine zeros • Completing the square and hence finding turning points • The discriminant and its significance for the number and nature of the zeros of a quadratic equation and the graph of a quadratic function • Using technology		✓	✓	✓*	✓*	AASL 2.7 (?presume technology approach taught also) AISL2.5(only use of technology to find roots)
		• The sum and product of the real zeros of a quadratic equation, and the associated algebra of surds		✓*?	✓			AASL1.7. AAHL 2.12.
		• Deducing quadratic models from the zeros and one other piece of data (e.g. another point), using suitable techniques and/or technologies • Understand the role of the discriminant		✓	✓	✓*	✓*	AASL 2.7 (presume technology approach taught in AA also) AISL2.5
	Subtopic 2.2: Cubic and quartic polynomials	• Leading coefficient • Degree			✓	✓*	✓*	AAHL2.12. AISL2.5(cubics only).
		Graphs of the cubic function in different forms.				✓*	✓*	AISL2.5 (modelling with cubics)
		• Cubics can be written as a product of a linear and a quadratic factor or as a product of three linear factors • What is the significance of these forms for the shape and number of zeros of the graph? • Cubic equations can be solved algebraically and by using technology			✓	✓*	✓*	AAHL2.12. AISL2.5 (modelling with cubics-using technology)
		Extending from the quadratic and cubic functions — behaviour can be expected from the graphs of quartic functions.			✓			AAHL2.12
Topic 3: Trigonometry		• Pythagoras' theorem • Trigonometric ratios						
		problems in contexts such as surveying, building, navigation, and design.		✓	✓	✓	✓	SL3.3 common content

		The cosine rule • Find the length of the third side when two sides and the included angle are known • Find the measure of an angle when the three sides are known		✓	✓	✓	✓	SL3.2 common content
		The sine rule • Find the measure of an unknown angle when two sides and the non-included angle are known • Find the length of one of the unknown sides where two angles and one side are known		✓	✓	✓*	✓*	SL3.2 common content(but not ambiguous case). AASL3.5 covers sine rule ambiguos case.
		contextual problems drawn from recreation and industry for an unknown side or angle.		✓	✓	✓	✓	SL3.2 common content
		find the area of a non-right triangle if the perpendicular to a side cannot be measured easily or accurately		✓	✓	✓	✓	SL3.2 common content
Subtopic 3.2: Circular measure and radian measure		graphs of cos θ, sinθ in degrees		✓	✓	✓	✓	AASL3.4(radians). AASL3.7(trig graphs). AISL2.5 (sinusoidal models only in degrees) AIHL2.9(sinusoidal models in radians) AIHL3.7(radians), AIHL3.8 (trig graphs).
		Unit circle and degrees cos θ, sinθ, tanθ and periodicity.		✓	✓		✓	AASL3.5. AIHL3.8
		• Define the radian measure of an angle as the length of the arc that subtends this angle at the centre of a unit circle • Apply the relationship to convert between radian and degree measure		✓	✓		✓	AASL3.4. AIHL3.7
		Calculate lengths of arcs and areas of sectors of circle.		✓	✓		✓	AASL3.4. AIHL3.7
Sub topic 3.3 Trigonometric functions		Unit circle and radians cos θ, sinθ, tanθ and periodicity.		✓	✓		✓	AASL3.5. AIHL3.8
		horizontal and vertical position of a point moving round a unit circle; the functions y - sinx and y = cosx.		✓	✓		✓	AASL3.5. AIHL3.8

		Recognising changes in amplitude, period, and phase		✓	✓	✓*	✓	AISL2.5(sinusoidal models in degrees only, and NO phase shift) AIHL2.9(sinusoidal models in radians) AIHL3.8(graphical methods of solving trigonometric equations inclusing radians) AASL3.7
		Identifying contexts suitable for modelling by trigonometric functions and use them to solve practical problems		✓	✓	✓*	✓	AASL3.7. AISL2.5(sinusoidal models in degrees only). AIHL2.9(sinusoidal models in radians)
		Solving trigonometric equations using technology and algebraically in simple cases		✓	✓		✓*	AASL3.8. AIHL3.8 (graphical methods only)
		examining the sine and cosine functions and their behaviour in the unit circle? • $\sin(-x) = -\sin x$ • $\cos(-x) = \cos x$ • $\sin(x + \frac{\pi}{2}) = \cos x$ • $\cos(x - \frac{\pi}{2}) = \sin x$		✓	✓			AASL3.5. AAHL3.11

		• Understanding the relationship between the angle of inclination and the gradient of the line
		• $\tan x = \frac{\sin x}{\cos x}$
		• The graphs of the functions
		  • $y = \tan x$
		  • $y = \tan Bx$
		  • $y = \tan(x + C)$
				✓	✓		✓	AASL3.7 AIHL3.8

Topic 4: Counting and statistics	**Subtopic 4.1: Counting**	number of ways something will occur be counted without listing all of the outcomes			✓			AAHL1.10
		• The multiplication principle • Factorials and factorial notation • Permutations			✓			AAHL1.10
		Understand the notion of a combination as an unordered set of distinct objects			✓			AAHL1.10

		The number of combinations (or selections) of r objects taken from a set of n distinct objects is C_r^n			✓			AAHL1.10
		Use $C_r^n = \frac{P_r^n}{r!} = \frac{n!}{(n-r)!r!}$ to solve problems. C_r^n.			✓			AAHL1.10
		Use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ for the number of combinations of r objects taken from a set of n distinct objects.			✓*			AAHL1.10 (notation isn't specified though) IB standard is the C^n_r notation
		Expand $(x+y)^n$ for integers $n = 1, 2, 3, 4$		✓	✓			AASL1.9
	Subtopic 4.2: Discrete and continuous random data	• Continuous variables may take any value (often within set limits); for example, height and mass • Discrete variables may take only specific values; for example, the number of eggs that can be purchased at a supermarket		✓	✓	✓	✓	SL4.1 common content. SL4.7(discrete random variables)
	Subtopic 4.3: Samples and statistical measures	Briefly consider mean, median, and mode		✓	✓	✓	✓	SL4.3 common content
		• Consider range and interquartile range • Standard deviation of a sample gives a useful measure of spread, which has the same units as the data		✓	✓	✓	✓	SL4.3 common content
		normal distributions		✓	✓	✓	✓	SL4.9 common content
		• Bell-shaped • Position of the mean • Symmetry about the mean • Characteristic spread • Unique position of one standard deviation from the mean		✓	✓	✓	✓	SL4.9 common content
		Variation in many quantities occurs in an approximately normal manner, and can be modelled using a normal distribution		✓	✓	✓	✓	SL4.9 common content - I assume as examples.
Topic 5: Growth and decay	**Subtopic 5.1: Indices and index laws**	Briefly consider indices (including negative and fractional indices) and the index laws.		✓	✓	✓*	✓*	SL1.5 common content does not cover fractional indices. AASL1.7 does.

		Define rational and irrational numbers and perform operations with surds and fractional indices	✓	✓	✓			AASL1.7
	Subtopic 5.2: Exponential functions	Establish and use the algebraic properties of exponential functions.		✓	✓	✓	✓	
		Recognise the qualitative features of the graph of $y=a^x (a>0)$ and of its translations $y=a^x+b$ and $y=a^{x-c}$ and dilation $y=ka^x$.		✓	✓	✓	✓	AASL2.9. AASL2.11. AISL2.5
		problems that involve exponential functions		✓	✓	✓	✓	AASL2.9. AASL2.11. AISL2.5
	Subtopic 5.3: Logarithmic functions	• Definition of the logarithm of a number • Rules for operating with logarithms $\log_a b = x \Leftrightarrow a^x = b$ and the relationships • $\log_a a^x = a^{\log_a x} = x$ • $\log_a mn = \log_a m + \log_a n$ • $\log_a \frac{m}{n} = \log_a m - \log_a n$ • $\log_a b^m = m \log_a b$		✓	✓	✓*	✓	SL1.5 common content has base 10 and e and the first rule for a general base a>0. AASL1.7. AIHL1.9 contain the remaining relationships.
		Solving exponential equations, using logarithms (base *10*)		✓	✓	✓	✓	SL1.5 common content
Topic 6: Introduction to differential calculus	**Subtopic 6.1: Rate of change**	rate of change		✓	✓	✓	✓	SL5.1 common content
		• The average rate of change of function $f(a)$ in the interval from a to $a+h$ is $\frac{f(a+h)-f(a)}{h}$ • The average rate of change is interpreted as the slope of a chord		✓	✓	✓	✓	Not explicitly mentioned but I assume it is covered in SL5.1 as the derivative is defined as the limit of the average rate of change.
	Subtopic 6.2: The concept of a derivative	• numerically from tables of data • algebraically from a formula • graphically (and geometrically) by considering gradients of chords across graphs of curves (graphics calculators, interactive geometry, and graphing software provide invaluable visual support, immediacy, and relevance for this concept).		✓	✓	✓	✓	Not explicitly mentioned but I assume it is covered in SL5.1 as the derivative is defined as the limit of the average rate of change.

		Rate of change at a point.		✓	✓	✓	✓	Not explicitly mentioned but I assume it is covered in SL5.1 as the derivative is defined as the limit of the average rate of change.
		The limit of the average rate of change over an interval that is approaching zero		✓	✓	✓	✓	Not explicitly mentioned but I assume it is covered in SL5.1 as the derivative is defined as the limit of the average rate of change.
		instantaneous rate of change using derivatives determined from first principles			✓			AAHL5.12 (first principles)
		Find the derivative function from first principles using $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. Introduce the alternative notation for the derivative of a function $f'(x)=\frac{\mathrm{d}y}{\mathrm{d}x}$.			✓			AAHL5.12 (first principles)
		Establish the formula $\frac{\mathrm{d}y}{\mathrm{d}x}=nx^{n-1}$ when $y=x^n$ where n is an integer.		✓	✓	✓		SL5.3 common content (not first principles) AAHL5.12 (first principles)
	Subtopic 6.4: Properties of derivatives	The use of differentiation by first principles for a number of examples of simple polynomials develops the rule $h'(x)=f'(x)\pm g'(x)$ for $h(x)=f(x)\pm g(x)$ which leads to $h'(x)=kf'(x)$ for $h(x)=kf(x)$.		✓	✓	✓		SL5.3 common content (not first principles) AAHL5.12 (first principles)
		Calculate derivatives of polynomials and other linear combinations of power functions		✓	✓	✓		SL5.3 common content (not first principles)
		Solve problems that use polynomials and other linear combinations of power functions, involving the following concepts: • The slope and equation of a tangent • Displacement and velocity • Rates of change • increasing and decreasing functions		✓	✓	✓*	✓	SL5.2 and SL5.4 common content but displacement and velocity are not mentioned. AASL5.9 has kinematics, ie displacement and velocity. AIHL5.13 has kinematics, ie displacement and velocity.

		• Maxima and minima, local and global
		  • stationary points
		  • sign diagram of the first derivative
		  • end points
				✓	✓	✓	✓	AASL5.8. AISL5.6

		Optimisation		✓*	✓*	✓*	✓*	AASL5.8 and AISL5.7 both cover optimisation, but not for kinematics.

Topic 7: Arithmetic and geometric sequences and series	**Subtopic 7.1: Arithmetic sequences and series**	Find the generative rule for a sequence, both recursive and explicit, using $t_{n+1} = t_n + d$ and $t_n = t_1 + (n-1)d$		✓	✓	✓	✓	SL1.2 common content
		· Determine the value of a term or the position of a term in a sequence		✓	✓	✓	✓	SL1.2 common content
		· Describe the nature of the growth observed		✓	✓	✓	✓	SL1.2 common content
		• $S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(2t_1 + (n-1)d)$		✓	✓	✓	✓	SL1.2 common content
	Subtopic 7.2: Geometric sequences and series	· Recognise and use the recursive definition of a geometric sequence: $t_{n+1} = rt_n$		✓	✓	✓	✓	SL1.3 common content
		Use the formula for the general form of a geometric sequence and recognise its exponential nature $t_n = r^{n-1}t_1$		✓	✓	✓	✓	SL1.3 common content
		Understand the limiting behaviour as n -> infinity of the terms and its dependence on the value of the common ratio *r*		✓	✓	✓	✓	SL1.3 common content
		• Establish and use the formula $S_n = t_1 \frac{r^n - 1}{r - 1}$ for the sum of the first n terms of a geometric sequence		✓	✓	✓	✓	SL1.3 common content
		Investigate the consequence of $\|r\| < 1$: $1 + r + r^2 + \ldots + r^n = \frac{1 - r^{n+1}}{1 - r}$,		✓	✓	✓	✓	SL1.3 common content
		• If $\|r\| < 1$ then $S_n \to \frac{t_1}{1 - r}$ as $n \to \infty$		✓	✓	✓	✓	SL1.3 common content

Topic 8: Geometry	**Subtopic 8.1: Circle properties**	• Chord and tangent properties ◆ Radius and tangent property ◆ Angle between tangent and chord (alternate segment theorem) ◆ Length of the two tangents from an external point						SACE Topic 8.1 not taught in IB
		• Properties of angles within circles ◆ Angle subtended at the centre is twice the angle subtended at the circumference by the same arc ◆ Angles at the circumference subtended by the same arc are equal ◆ Opposite angles in a cyclic quadrilateral are supplementary ◆ An angle in a semicircle is a right angle ◆ Chords of equal length subtend equal angles at the centre ◆ Converses of the above properties ◆ Intersecting chords theorem, including internal and external intersections, and the special case of a tangent and chord through an external point						SACE Topic 8.1 not taught in IB
	Subtopic 8.2: The nature of proof	Justification of properties of circles						SACE Topic 8.1 not taught in IB
		The nature of proof: • Note that use of similarity and congruence is required in some proofs • Use implication, converse, equivalence, negation, contrapositive • Use examples and counter-examples • Use proof by contradiction.	✓?	✓*	✓			Only simple examples would be covered in SACE Stage 1 Mathematics AASL1.6 simple deductive proofs. AAHL1.15(proof by contradiction, use of a counter example to show that a statement is not true)
Topic 9: Vectors in the plane	**Subtopic 9.1: Vector operations**	Representing vectors in the plane by directed line segments			✓		✓	AAHL3.12. AIHL3.10
		• Vector addition and subtraction • Scalar multiples of a vector • Applications of scalar multiples: parallel vectors, ratio of division			✓		✓	AAHL3.12. AIHL3.10

	Subtopic 9.2: Component and unit vector forms	• Use ordered-pair notation and column vector notation • Convert a vector into component and unit vector forms • Determine length and direction of a vector from its components			✓		✓	AAHL3.12. AIHL3.10
	Subtopic 9.3: Projections	In this subtopic, students work out the projection of one vector onto another.						
		• The dot (scalar) product • The angle between two vectors • Perpendicular vectors • Parallel vectors			✓		✓	AAHL3.13. AIHL3.13
	Subtopic 9.4: Geometric proofs using vectors	Geometric proofs using vectors in the plane include: • The diagonals of a parallelogram meet at right angles if and only if it is a rhombus • Midpoints of the sides of a quadrilateral join to form a parallelogram • The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides			✓?		✓?	??? I assume these are included in both AAHL and AIHL ???
Topic 10: Further trigonometry	**Subtopic 10.1: Further trigonometric functions**	Using graphing technology, students can explore the effects of the four control numbers (individually and in combination) in the general sinusoidal model $y = A\sin B(x-C)+D$ on transforming the graph of $y=\sin x$. Students explore fitting functions of this form to their data		✓	✓		✓	AASL3.7. AIHL2.9
		• The general function $y = A\sin B\left(x-C\right)+D$ • Extend to: $y = A\cos B\left(x-C\right)+D$ $y = A\tan B\left(x-C\right)+D$		✓	✓		✓	AASL3.7. AIHL2.9
		Sketch graphs of sinusoidal functions		✓	✓		✓	AASL3.7. AIHL2.9
		• Solve trigonometric equations of the form $y=k$ (where y is one of the functions above), finding all solutions		✓	✓		✓	AASL3.8 AIHL3.8 (graphical methods emphasised)

	Subtopic 10.2: Trigonometric identities	Students are guided through the deduction of many of these useful identities by looking at the unit circle. They discover others by comparing their graphs. The formula for $\cos(A-B)$ can be derived from the unit circle, using the cosine rule. The other angle sum formulae follow from it, using the identities already learnt.						
		• $\sin(-x), \cos(-x)$ $\sin^2 x + \cos^2 x,\ \sin 2x$ $\cos 2x,\ \sin\frac{1}{2}x,\ \cos\frac{1}{2}x$ in terms of $\sin x,\ \cos x$ • $\cos(A \pm B)$, and hence $\sin(A \pm B)$ in terms of $\sin A,\ \cos A,\ \sin B,\ \cos B$		✓*	✓			AASL3.5, AAHL3.9. AAHL3.10
		• Conversion of $A\sin x + B\cos x$ into the form $k\sin(x+\alpha)$						Not explicitly mentioned in the IB guides
		• The reciprocal trigonometric functions: $\sec\theta,\ \operatorname{cosec}\theta,\ \cot\theta$			✓			AAHL3.9
Topic 11: Matrices	**Subtopic 11.1: Matrix arithmetic**	What is a matrix? • Order of matrices What operations can be applied to matrices? • Addition and subtraction • Scalar multiplication • Matrix multiplication					✓	AIHL1.14
		The identity matrix for matrix multiplication What is the inverse of a square matrix?					✓	AIHL1.14
		The formula $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ for a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ The number $ad - bc$ is called the determinant of A and is denoted $\det A$. If $\det A = 0$, the inverse does not exist.					✓	AIHL1.14

		How can matrix inverses be used? • Find the unique solution to matrix equations of the form $AX = B$ or $XA = B$ if it exists					✓	AIHL1.14
	Subtopic 11.2: Transformations in the plane	Transformations in the plane and their description in terms of matrices. • Translations and their representation as column vectors, that is, 2×1 matrices • Define and use basic linear transformations					✓	AIHL3.9
		• Consider dilations of the form $(x, y)\rightarrow(\lambda_1 x, \lambda_2 y)$, rotations about the origin where $\begin{bmatrix} x \\ y \end{bmatrix}\rightarrow\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, and reflection in a line which passes through the origin where $\begin{bmatrix} x \\ y \end{bmatrix}\rightarrow\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$					✓	AIHL3.9
		• Apply transformations to points in the plane and geometric objects • Define and use composition of linear transformations and the corresponding matrix products					✓	AIHL3.9
		• Establish geometric results by matrix multiplications • Show that the combined effect of two reflections in lines through the origin is a rotation					✓?	Not explicitly mentioned but I assume this is in AIHL3.9
		• Define and use inverses of linear transformations and the relationship with the matrix inverse • Examine the relationship between the determinant and the effect of a linear transformation on area • Note that if the determinant of a matrix is zero, then the corresponding transformation has no inverse					✓?	Not explicitly mentioned but I assume this is in AIHL3.9
Topic 12: Real and complex numbers	**Subtopic 12.1: The number line**	The number line represents all real numbers. What are some properties of special subsets of the reals? • Rational and irrational numbers	✓					

		Consider surds and their operations: • Express rational numbers as terminating or eventually recurring decimals and vice versa. • Prove irrationality by contradiction for numbers such as $\sqrt{2}$ and $\log_2 5$.	✓	✓*	✓			AASL1.6 simple deductive proofs. AAHL1.15(proof by contradiction, use of a counter example to show that a statement is not true)
		• Proving simple results involving numbers Some examples include: • The sum of two odd numbers is even. • The product of two odd numbers is odd. • The sum of two rational numbers is rational.		✓	✓			AASL1.6
		• Interval notation Use of square brackets and parentheses to denote intervals of the number line that include or exclude the endpoints. For example, the set of numbers x such that $a < x \leq b$ is denoted $(a, b]$.	✓					Note: Interval notation is different in the IB compared to SACE.
	Subtopic 12.2: Introduction to mathematical induction	Formal proofs of simple examples are expected, i.e. Let there be associated with each positive integer n, a proposition $P(n)$. If $P(1)$ is true, and for all k, $P(k)$ is true implies $P(k+1)$ is true, then $P(n)$ is true for all positive integers n. • prove results for simple sums, such as $1+4+9+...+n^2 = \frac{n(n+1)(2n+1)}{6}$ for any positive integer n • prove results for arithmetic and geometric series.			✓			AAHL1.15
	Subtopic 12.3: Complex numbers	Define the imaginary number i as a solution to the quadratic equation $x^2+1=0$. • Use complex numbers in the form $a+b\mathrm{i}$ where a and b are the real and imaginary parts. • Determine and use complex conjugates; for $z=a+b\mathrm{i}$, $\bar{z}=a-b\mathrm{i}$. • Perform complex-number arithmetic: addition, subtraction, multiplication, and division. Students can add and subtract complex numbers using the usual rules of arithmetic and algebra.			✓		✓	AAHL1.12, AAHL1.13. AIHL1.12

		Multiplication calls for the same approach, with the additional need to simplify i^2 using the fact that $\mathrm{i}^2 = -1$. Division of complex numbers can be introduced by presenting a complex product such as $(2-\mathrm{i})(1+\mathrm{i}) = 3+\mathrm{i}$, inferring the result for $\frac{3+\mathrm{i}}{2-\mathrm{i}}$			✓		✓	AAHL1.12, AAHL1.13. AIHL1.12
	Subtopic 12.4: The complex (Argand) plane	The Cartesian plane as extension of the real number line to two dimensions. Correspondence between the complex number $a+b\mathrm{i}$, the coordinates (a, b) and the vector $[a, b]$. Complex-number addition corresponds to vector addition via the parallelogram rule. Complex numbers can be regarded as two-dimensional vectors with rules for multiplication and division to supplement addition and subtraction. Relative positions of $z = a + b\mathrm{i}$ and its conjugate; their sum is real and difference purely imaginary. Recognising that $\lvert z \rvert = \sqrt{a^2 + b^2}$ represents the length of a complex number when represented as a vector.			✓		✓	AAHL1.12, AAHL1.13. AIHL1.12
	Subtopic 12.5: Roots of equations	• The introduction of i enables the solution of all real quadratic equations and the factorisation of all quadratic polynomials into linear factors			✓		✓	AAHL1.14. AIHL1.12
		When the solutions of a real quadratic equation are complex, they are conjugates.			✓		✓	AAHL1.14. AIHL1.12
✓*	partially included							
✓	included							

Stage 2 Mathematical Methods	Unit (sub topic)	Content	Included in Prior learning	Included in Analysis SL	Included in Analysis HL	Included in Apps SL	Included in Apps HL	Comments
Topic 1. Further differentiation and applications	**1.1 Introductory differential calculus**	What is the derivative of x^n, where n is a rational number? Establish the formula $\frac{\mathrm{d}y}{\mathrm{d}x} = nx^{n-1}$ from first principles when $y = x^n$, where n is a rational number Determine the derivative of linear combinations of power functions involving rational exponents		✓	✓	✓*	✓	SLAI - integer exponents only
		The derivative of a function can be used to find the slope of tangents to the function, and hence the equation of the tangent and/or normal at any point on the function		✓	✓	✓	✓	
		When an object's displacement is described by a function, the derivative can be used to find the instantaneous velocity		✓	✓		✓	
		The sign diagram of the derivative function can be used to find when the function is increasing, decreasing, and stationary		✓	✓	✓	✓	
		The derivative of a function can be used to determine the rate of change, and the position of any local maxima or minima		✓	✓	✓	✓	
	1.2 Differentiation rules	Functions can be classified as sums, products,		✓	✓		✓	
		Chain rule		✓	✓		✓	
		Product rule		✓	✓		✓	
		Quotient		✓	✓		✓	
	1.3 Exponential functions	The derivative of $y = ab^x$ is a multiple of the original function		✓	✓		✓	
		There exists an irrational number e so that $\frac{\mathrm{d}y}{\mathrm{d}x} = y = e^x$ The approximate value of e is 2.7182818		✓	✓		✓	
		Many exponential functions show growth or decay. This growth/decay may be unlimited or asymptotic to specific values		✓	✓	✓	✓	

		find the slope of tangents to the graphs of exponential functions, and hence the equation of the tangent and/or normal at any point on the function		✓	✓		✓	
		find the instantaneous velocity, when an object's displacement is described by an exponential function		✓	✓		✓	
		determine the rate of change, the position of any local maxima or minima, and when the function is increasing or decreasing		✓	✓		✓	
		Use exponential functions and their derivatives to solve practical problems where exponential functions model actual examples		✓	✓		✓	
	1.4 Trigonometric functions	Graphing sine and cosine functions in Topic 3: Trigonometry, Stage 1 Mathematics introduced the concept of radian angle measure, the use of sine and cosine to define different aspects of the position of a moving point on a unit circle, and the ability to solve trigonometric equations		✓	✓	✓*	✓	SLAI - using degrees only
		When t is a variable measured in radians (often time), sin(t) and cos(t) are periodic functions		✓	✓		✓	
		$y = \sin t$ has a derivative $\frac{\mathrm{d}y}{\mathrm{d}t} = \cos t$ $y = \cos t$ has a derivative $\frac{\mathrm{d}y}{\mathrm{d}t} = -\sin t$		✓	✓		✓	
		The use of the quotient rule on $\frac{\sin t}{\cos t}$ allows the derivative of $\tan t$ to be found Derivatives can be found for functions such as $x\sin x$, $\frac{e^x}{\cos x}$ The chain rule can be applied to $\sin f(x)$ and $\cos f(x)$		✓	✓		✓	

		find the slope of tangents to the graphs of trigonometric functions, and hence the equation of the tangent and/or normal at any point on the function		✓	✓		✓	
		find the instantaneous velocity, when an object's displacement is described by a trigonometric function		✓	✓		✓	
		determine the rate of change, the position of any local maxima or minima, and when the function is increasing or decreasing		✓	✓		✓	
		Use trigonometric functions and their derivatives to solve practical problems where trigonometric functions model periodic phenomena		✓	✓		✓	
	1.5 The second derivative	The second derivative is the result of differentiating the derivative of a function		✓	✓		✓	
		The notations y'', $f''(x)$ and $\frac{d^2y}{dx^2}$ can be used for the second derivative		✓	✓		✓	
		The second derivative of a displacement function describes the acceleration of a particle, and is used to determine when the velocity is increasing or decreasing		✓	✓		✓	
		The sign diagram of the first and second derivative provides information to assist in sketching the graphs of functions		✓	✓		✓	
		Stationary points occur when the first derivative is equal to zero, and may be local maxima, local minima, or stationary inflections		✓	✓	✓	✓	
		Points of inflection occur when the second derivative equals zero and changes sign, and may be classed as stationary or non stationary		✓	✓		✓	
		The second derivative can be used to describe the concavity of a curve		✓	✓		✓	

		Whether the second derivative is positive, negative, or zero at a stationary point is used to determine the nature of the stationary point		✓	✓		✓	
Topic 2. Discrete random variables	**2.1 Discrete random variables**	A random variable is a variable, the value of which is determined by a process, the outcome of which is open to chance. For each random variable, once the probability for each value is determined it remains constant		✓	✓	✓	✓	
		Continuous random variables may take any value (often within set limits)		✓	✓	✓	✓	
		Discrete random variables may take only specific values		✓	✓	✓	✓	
		A probability function specifies the probabilities for each possible value of a discrete random variable. This collection of probabilities is known as a probability distribution		✓	✓	✓	✓	
		A table or probability bar chart can show the different values and their associated probability		✓	✓	✓	✓	
		The sum of the probabilities must be 1		✓	✓	✓	✓	
		probabilities for the different outcomes are different, whereas uniform discrete random variables have the same probability for each outcome		✓	✓	✓	✓	HLAA and HLAI includes includes the effect of linear transformations of X
		When a large number of independent trials is considered, the relative frequency of an event gives an approximation for the probability of that event		✓	✓	✓	✓	
		The expected value of a discrete random variable is calculated using $E(X)=\sum xp(x)=\mu_X$, where $p(x)$ is the probability function for achieving result x and μ_X is the mean of the distribution		✓	✓	✓	✓	
		The principal purpose of the expected value is to be a measurement of the centre of the distribution		✓	✓	✓	✓	

		The expected value can be interpreted as a long-run sample mean		✓	✓	✓	✓	
		The standard deviation of a discrete random variable is calculated $\sigma_X = \sqrt{\sum[x-\mu_X]^2 p(x)}$ where μ_X is the expected value and $p(x)$ is the probability function for achieving result x			✓			
		The principal purpose of the standard deviation is to be a measurement of the spread of the distribution			✓			
	2.2 The Bernoulli distribution	Discrete random variables with only two outcomes are called Bernoulli random variables. These two outcomes are often labelled ‘success’ and ‘failure’		✓	✓	✓	✓	
		The Bernoulli distribution is the possible values and their probabilities of a Bernoulli random variable		✓	✓	✓	✓	
		One parameter, p, the probability of ‘success’, is used to describe Bernoulli distributions		✓	✓	✓	✓	
		The mean of the Bernoulli distribution is p, and the standard deviation is given by $\sqrt{p(1-p)}$		✓	✓	✓	✓	
	2.3 Repeated Bernoulli trials and the binomial distribution	When a Bernoulli trial is repeated, the number of successes is classed as a binomial random variable		✓	✓	✓	✓	
		The possible values for the different numbers of successes and their probabilities make up a binomial distribution		✓	✓	✓	✓	
		The mean of the binomial distribution is np, and the standard deviation is given by $\sqrt{np(1-p)}$, where p is the probability of success in a Bernoulli trial and n is the number of trials		✓	✓	✓	✓	
		A binomial distribution is suitable when the number of trials is fixed in advance, the trials are independent, and each trial has the same probability of success		✓	✓	✓	✓	

		The probability of k successes from n trials is given by $\Pr(X=k)=C_k^n p^k (1-p)^{n-k}$, where p is the probability of success in the single Bernoulli trial		✓	✓	✓	✓	
		Students, given the probability of success, calculate probabilities such as: • exactly k successes out of n trials • at least k successes out of n trials • between k_1 and k_2 successes out of n trials		✓	✓	✓	✓	
		The binomial distribution for large values of n has a symmetrical shape that many students will recognise				✓	✓	
Topic 3. Integral calculus	**3.1 Anti-differentiation**	Finding a function whose derivative is the given function is called 'anti-differentiation'		✓	✓	✓	✓	
		Anti-differentiation is more commonly called 'integration' or 'finding the indefinite integral'		✓	✓	✓	✓	
		Any function $F(x)$ such that $F'(x)=f(x)$ is called the indefinite integral of $f(x)$ The notation used for determining the indefinite integral is $\int f(x)\mathrm{d}x$		✓	✓	✓	✓	
		All families of functions of the form $F(x)+c$ for any constant c have the same derivative. Hence, if $F(x)$ is an indefinite integral of $f(x)$, then so is $F(x)+c$ for any constant c		✓	✓	✓	✓	
		By reversing the differentiation processes, the integrals of x^n (for $n \neq -1$), e^x, $\sin x$, and $\cos x$ can be determined		✓	✓	✓*	✓	SLAI polynomial functions only
		Reversing the differentiation processes and consideration of the chain rule can be used to determine the integrals of $[f(x)]^n$ (for $n \neq -1$), $e^{f(x)}$, $\sin f(x)$, and $\cos f(x)$ for linear functions $f(x)$ $\int[f(x)+g(x)]\mathrm{d}x=\int f(x)\mathrm{d}x+\int g(x)\mathrm{d}x$		✓	✓		✓	

		When the value of the indefinite integral is known for a specific value of the variable (often an initial condition), the constant of integration can be determined		✓	✓	✓	✓	
		Determine the displacement given the velocity in a linear motion problem		✓	✓		✓	
	3.2 Area under curves	The area under a simple positive monotonic curve is approximated by upper and lower sums of the areas of rectangles of equal width		✓	✓	✓	✓	
		Decreasing the width of the rectangles improves the estimate of the area, but makes it more cumbersome to calculate		✓	✓	✓	✓	
		The exact value of the area is the unique number between the upper and lower sums, which is obtained as the width of the rectangles approaches zero		✓	✓	✓	✓	
		The definite integral $\int_a^b f(x)\,\mathrm{d}x$ can be interpreted as the exact area of the region between the curve $y=f(x)$ and the x-axis over the interval $a \le x \le b$ (for a positive continuous function $f(x)$)		✓	✓	✓	✓	
		When $f(x)$ is a continuous negative function the exact area of the region between the curve $y=f(x)$ and the x-axis over the interval $a \le x \le b$ is given by: $-\int_a^b f(x)\,\mathrm{d}x$		✓	✓		✓	
		When $f(x)$ is above $g(x)$, that is $f(x) \ge g(x)$, the area is given by $\int_a^b [f(x)-g(x)]\,\mathrm{d}x$		✓	✓			
	3.3 Fundamental theorem of calculus	• The statement of the fundamental theorem of calculus $$\int_a^b f(x)\,\mathrm{d}x = F(b)-F(a)$$ where $F(x)$ is such that $F'(x)=f(x)$		✓	✓	✓	✓	

		$\int_a^a f(x)\mathrm{d}x = 0$ $\int_a^b f(x)\mathrm{d}x + \int_b^c f(x)\mathrm{d}x = \int_a^c f(x)\mathrm{d}x$		✓	✓	✓	✓	
		In the exploration of areas in Subtopic 3.2, the use of technology meant that the exact value of areas (or the exact value of one of the end points of the area) could not always be obtained. The fundamental theorem of calculus can be used in those circumstances		✓	✓		✓	
		Applications can be modelled by functions, and evaluating the area under or between curves can be used to solve problems		✓	✓		✓	
		When the rate of change of a quantity is graphed against the elapsed time, the area under the curve is the total change in the quantity		✓	✓		✓	
		The total distance travelled by an object is determined from its velocity function		✓	✓		✓	
		An object's position is determined from its velocity function if the initial position (or position at some specific time) is known		✓	✓		✓	
		An object's velocity is determined from its acceleration function if the initial velocity (or velocity at some specific time) is known		✓	✓		✓	
Topic 4. Logarithmic functions	**4.1 Using logarithms for solving exponential equations**	The solution for x of the exponential equation $a^x = b$ is given using logarithms $x = \log_a b$		✓	✓		✓	
		The natural logarithm is the logarithm of base *e*		✓	✓		✓	

		When $y = e^x$ then $x = \log_e y = \ln y$ Natural logarithms obey the laws: $\ln a^b = b \ln a$ $\ln ab = \ln a + \ln b$ $\ln \frac{a}{b} = \ln a - \ln b$		✓	✓		✓	
		find the slope of tangents to the graphs of logarithmic functions, and hence the equation of the tangent and/or normal at any point on the function		✓	✓		✓	
		find the instantaneous velocity, when an object's displacement is described by an exponential function		✓	✓		✓	
		determine the rate of change, the position of any local maxima or minima, and when the function is increasing or decreasing		✓	✓		✓	
	4.2: Logarithmic functions and their graphs	In many areas of measurement, a logarithmic scale is used to render an exponential scale linear or because the numbers cover too large a range to make them easy to use					✓	
		The graph of $y = \ln x$ is continuously increasing, with an x-intercept at $x = 1$ and a vertical asymptote $x = 0$		✓	✓		✓	
		The graph of $y = k \ln(b(x-c))$ is the same shape as the graph of $y = \ln x$, with the values of k, b, and c determining its specific characteristics		✓	✓		✓	
		Like all inverse functions, the graphs of $y = e^x$ and $y = \ln x$ are reflections of each other in the line $y = x$		✓	✓		✓	
	4.3: Calculus of logarithmic functions	• The function $y = \ln x$ has a derivative $\frac{dy}{dx} = \frac{1}{x}$ • The function $y = \ln f(x)$ has a derivative $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$		✓	✓		✓	
		Provided x is positive, $\int \frac{1}{x} dx = \ln x + c$		✓	✓		✓	

		find the slope of tangents to the function		✓	✓		✓	
		find the instantaneous velocity, when an object's displacement is described by a logarithmic function		✓	✓		✓	
		determine the rate of change, the position of any local maxima or minima, and when the function is increasing or decreasing		✓	✓		✓	
		Use logarithmic functions and their derivatives to solve practical problems.		✓	✓		✓	
Topic 5: Continuous random variables and the normal distribution	**5.1: Continuous random variables**	A continuous random variable can take any value (sometimes within set limits)		✓	✓			
		The probability of each specific value of a continuous random variable is effectively zero. The probabilities associated with a specific range of values for a continuous random variable can be estimated from relative frequencies and from histograms			✓			
		A probability density function is a function that describes the relative likelihood for the continuous random variable to be a given value			✓			HLAA and HLAI includes includes the effect of linear transformations of X
		A function is only suitable to be a probability density function if it is continuous and positive over the domain of the variable. Additionally, the area bound by the curve of the density function and the *x* -axis must equal 1, when calculated over the domain of the variable			✓			
		The area under the probability density function from *a* to *b* gives the probability that the values of the continuous random variable are between *a* and *b*			✓			

		the mean: $$\mu_X = \int_{-\infty}^{\infty} x\, f(x)\,\mathrm{d}x$$ the standard deviation: $$\sigma_X = \sqrt{\int_{-\infty}^{\infty} [x-\mu_X]^2 f(x)\,\mathrm{d}x}$$			✓			HLAA also includes mode and median
	5.2: Normal distributions	Natural variation means that many random variables have the majority of values close to the mean, with symmetrically lower probabilities as the values get further above or below the mean. Under some conditions these variables can be classed as 'normal random variables'		✓	✓	✓	✓	
		The normal distribution is symmetric and bell-shaped. Each normal distribution is determined by the mean μ and the standard deviation σ		✓	✓	✓	✓	
		$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$		✓	✓	✓	✓	
		When the normal distribution is used to model data, the area under the curve within an interval is interpreted as a proportion or a probability		✓	✓	✓	✓	
		When one limit of a known area (upper or lower) is known, the other can be obtained		✓	✓	✓	✓	
		All normal distributions can be transformed to the standard normal distribution with $\mu = 0$ and $\sigma = 1$ by using the formula: $$Z = \frac{X-\mu}{\sigma}$$		✓	✓	✓	✓	
	5.3: Sampling	For n independent observations of X, the sum of the observations $(X_1 + X_2 + X_3 + \ldots + X_n)$ is a random variable S_n The distribution of S_n is called its sampling distribution The sampling distribution of S_n has mean $n\mu$ and standard deviation $\sigma\sqrt{n}$					✓	

		For n independent observations of X, the sample mean of the observations $\left(\frac{X_1+X_2+X_3+\ldots+X_n}{n}\right)$ is a random variable $\overline{X}_n$ The distribution of $\overline{X}_n$ is called its sampling distribution The distribution of the random variable $\overline{X}_n$ has a mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$					✓	
		For any value of n, S_n is normally distributed, with mean $n\mu$ and standard deviation $\sigma\sqrt{n}$					✓	
		For any value of n, $\overline{X}_n$ is normally distributed, with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$					✓	
		Provided n is sufficiently large, S_n is approximately normally distributed, with mean $n\mu$ and standard deviation $\sigma\sqrt{n}$ Provided n is sufficiently large, $\overline{X}_n$ is approximately normally distributed, with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$ ıe central limit theorem.					✓	
		A simple random sample of size *n* is a collection of *n* subjects chosen from a population in such a way that every possible sample of size *n* has an equal chance of being selected					✓	
		If one simple random sample of n individuals is chosen from a population, and the value of a certain variable X is recorded for each individual in the sample, the sample mean of the n values for this one sample $\bar{x}$ is one observation of the random variable denoted $\overline{X}_n$					✓	

		If X has population mean μ and population standard deviation σ, then the sampling distribution of $\overline{X}_n$ is approximately normal with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$ provided that the sample size n is sufficiently large but also small compared with the population size N Approximate probabilities for $\overline{X}_n$ can be calculated from the sampling distribution					✓	
Topic 6: Sampling and confidence intervals	**6.1: Confidence intervals for a population mean**	If a sample is chosen and its mean calculated then the value of that sample mean will be variable. Different samples will yield different sample means					✓	
		Sample means are continuous random variables					✓	
		For a sufficiently large sample size, the distribution of sample means will be approximately normal. The distribution of sample means will have a mean equal to μ, the population mean. This distribution has a standard deviation equal to $\frac{\sigma}{\sqrt{n}}$, where σ is the standard deviation of the population and n is the sample size					✓	
		An interval can be created around the sample mean that will be expected, with some specific confidence level, to contain the population mean					✓	
		• If $\bar{x}$ is the sample mean and s the standard deviation of a suitably large sample, then the interval: $\bar{x} - z\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + z\frac{s}{\sqrt{n}}$ can be created. The value of z is determined by the confidence level required					✓	
		Not all confidence intervals will contain the true population mean					✓	
		The inclusion or not of a claimed population mean within a confidence interval can be used as a guide to whether the claim is true or false, but definitive statements are not possible					✓	

	6.2: Population proportions	A population proportion p is the proportion of elements in a population that have a given characteristic. It is usually given as a decimal or fraction					✓	
		A population proportion represents the probability that one element of the population, chosen at random, has the given characteristic being studied					✓	
		If a sample of size n is chosen, and X is the number of elements with a given characteristic, then the sample proportion $\hat{p}$ is equal to $\frac{X}{n}$					✓	
		A sample proportion is a discrete random variable. The distribution has a mean of p and a standard deviation of $\sqrt{\frac{p(1-p)}{n}}$					✓	
		As the sample size increases, the distribution of $\hat{p}$ becomes more and more like a normal distribution					✓	
		An interval can be created around the sample proportion that will be expected, with some specific confidence level, to contain the population proportion					✓	
		If $\hat{p}$ is the sample proportion, then the interval $\hat{p}-z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p}+z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ can be created. The value of z is determined by the confidence level required					✓	
		Not all confidence intervals will contain the true population proportion					✓	
		The inclusion or not of a claimed population proportion within a confidence interval can be used a guide to whether the claim is true or false, but definitive statements are not possible					✓	
✓*	partially included							
✓	included							

Stage 2 Specialist Mathematics	Unit (Topic)	Content	Included in Prior learning	Included in Analysis SL	Included in Analysis HL	Included in Apps SL	Included in Apps HL	Comments
Topic 1: Mathematical induction	**Subtopic 1.1: Proof by mathematical induction**	the nature of inductive proof, including the initial statement and inductive step			✓			
		divisibility, sums, products, trigonometry, matrices, complex numbers.			✓			
Topic 2: Complex numbers	**Subtopic 2.1: Cartesian and polar forms**	• real and imaginary parts, $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$, of a complex number • Cartesian form • arithmetic using Cartesian forms			✓		✓	
		Consider describing sets of points in the complex plane, such as circular regions or regions bounded by rays from the origin			✓		✓	
		Conversion between Cartesian form and polar form			✓		✓	Euler form not in Specialist Maths
		• The properties $\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$ $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$ $\mathrm{cis}\,\theta_1 . \mathrm{cis}\,\theta_2 = \mathrm{cis}(\theta_1 + \theta_2)$ $\dfrac{\mathrm{cis}\,\theta_1}{\mathrm{cis}\,\theta_2} = \mathrm{cis}(\theta_1 - \theta_2)$ They are the basis on which multiplication by $r\,\mathrm{cis}\,\theta$ is interpreted as dilation by r and rotation by θ			✓		✓	
		The utility of the polar form in calculating multiplication, division, and powers of complex numbers, and the geometrical interpretation of these			✓		✓	
		Prove and use de Moivre's theorem			✓		✓*	AIHL1.13 mentions integer powers of complex numbers in polar form, but De Moivre is not specifically mentioned.

		Extension to negative integral powers and fractional powers			✓		✓*	
	Subtopic 2.2: The complex (Argand) plane	Examine and use addition of complex numbers as vector addition in the complex plane			✓		✓*	
		Examine and use multiplication as a linear transformation in the complex plane			✓		✓	not stated explicitly in AA or AI HL, but assuming would be part of the teaching
		Use multiplication by *i* as anticlockwise rotation through a right angle			✓		✓	not stated explicitly in AA or AI HL, but assuming would be part of the teaching
		Apply the geometric notion of $\lvert z-w\rvert$ as the distance between points in the plane representing them			✓			not stated explicitly in AA or AI HL, but assuming would be part of the teaching
		Investigate the triangle inequality						not explicitly stated in AA or AI HL courses.
		Apply geometrical interpretation and solution of equations describing circles, lines, rays, and inequalities describing associated regions; obtaining equivalent Cartesian equations and inequalities where appropriate			✓			not stated explicitly in AA or AI HL, but assuming wouold be part of the teaching
	Subtopic 2.3: Roots of complex numbers	Solution of $z^n = c$ for complex c but in particular the case $c = 1$			✓			
		Finding solution of n^{th} roots of complex numbers and their location in the complex plane			✓			
	Subtopic 2.4: Factorisation of polynomials	the division algorithm using long division or synthetic division, or the multiplication process with inspection.						Pretty sure this would be taught in IB but can't find explicit reference to it.
		equating coefficients in factoring when one factor is given.						Pretty sure this would be taught in IB but can't find explicit reference to it.
		• Consider roots, zeros, and factors • Prove and apply the factor and remainder theorem; its use in verifying zeros			✓			In Specialist Maths: sum and product of roots of polynomials only for quadratics.
		Consider conjugate roots in factorisation of cubics and quartics with real coefficients (given a zero)			✓			
		Solve simple real polynomial equations			✓			

Topic 3: Functions and sketching graphs	**Subtopic 3.1: Composition of functions**	• If f and g are two functions, then the composition function $(f \circ g)(x)$ is defined by $f(g(x))$ if this exists		✓	✓		✓	
		Determine when the composition $(f \circ g)(x) = f(g(x))$ of two functions is defined		✓	✓		✓	
		Find compositions		✓	✓		✓	
	Subtopic 3.2: One-to-one functions	Determine if a function is one-to-one			✓		✓	
		Determine the inverse of a one-to-one function			✓		✓	
		Relationship between the graph of a function and the graph of its inverse • Investigate symmetry about $y = x$		✓	✓	✓	✓	
	Subtopic 3.3: Sketching graphs	Absolute value function and its properties			✓			
		Compositions involving absolute values and reciprocals			✓			Solving modulus equations and inequalities not in Specialist Maths
		Graphs of rational functions		✓	✓			
Topic 4: Vectors in three dimensions	**Subtopic 4.1: The algebra of vectors in three dimensions**	Develop 2D vectors to 3D		✓	✓		✓	
	Subtopic 4.2: Vector and Cartesian equations	Introduce Cartesian coordinates by plotting points and considering relationships between them. Two and three D: Consider vector, parametric, and Cartesian forms. Compute the point of a given line that is closest to a given point; distance between skew lines; and angle between lines		✓	✓	✓ *	✓	No Volume of 3D shapes, no angle between two planes in Specialist Maths
		Scalar (dot) product and vector (cross) product. Interpret them in context			✓		✓	
		Perform cross-product calculation using the determinant to determine a vector normal to a given plane			✓		✓	

		$\lvert \boldsymbol{a} \times \boldsymbol{b} \rvert$ is the area of the parallelogram with sides $\boldsymbol{a}$ and $\boldsymbol{b}$			✓		✓	
		Explore the following relationships: intersection of a line and a plane, angle between a line and a plane, and lines parallel to or coincident with planes. Derive the equation of a plane in Cartesian form, Ax +By +Cz + D = 0		✓ *	✓			
		$\frac{\lvert Ax_1 + By_1 + Cz_1 + D \rvert}{\sqrt{A^2 + B^2 + C^2}}$. The shortest distance between a point (x1, y1, z1) to a plane			✓			
		Prove geometric results in the plane and construct simple proofs in three dimensions: • Equality of vectors • Coordinate systems and position vectors; components • The triangle inequality			✓			
		The use of vector methods of proof, particularly in establishing parallelism, perpendicularity, and properties of intersections			✓			
	Subtopic 4.3: Systems of linear equations	form of a system of linear equations in several variables, and use elementary techniques of elimination (row operations) on augmented matrix form to solve a system of up to 3x3 linear equations 3×3			✓			
		Discuss intersections of planes: algebraic and geometric descriptions of unique solution, no solution, and infinitely many solutions.		✓	✓			
		Finding the intersection of a set of two or more planes amounts to solving a system of linear equations in three unknowns.		✓	✓			

Topic 5: Integration techniques and applications	**Subtopic 5.1: Integration techniques**	Use identities to simplify integrals of squared trigonometric functions		✓	✓		✓	
		Use substitution $u = g(x)$ to integrate expressions of the form $f(g(x))g'(x)$		✓	✓		✓	
		Use the formula $\int \frac{1}{x} \mathrm{d}x = \ln\lvert x \rvert + c$ for $x \neq 0$		✓	✓		✓	
		Find and use the inverse trigonometric functions: arcsine, arccosine, arctangent			✓			
		• Find and use the derivatives of these functions • Hence integrate expressions of the form $\frac{\pm 1}{\sqrt{a^2 - x^2}}$, $\frac{a}{a^2 + x^2}$			✓			
		Use partial fractions for integrating rational functions in simple cases			✓			
		Use integration by parts			✓			repeated by parts not in Specialist Maths
	Subtopic 5.2: Applications of integral calculus	Areas between curves determined by functions		✓	✓		✓	
		Volumes of solids of revolution about either axis			✓		✓	
Topic 6: Rates of change and differential equations	**Subtopic 6.1: Implicit differentiation**	Implicit differentiation			✓			
	Subtopic 6.2: Differential equations	Related rates			✓			
		• Solve differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)$ • Solve differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y)$			✓		✓	Numerical solution of first order DE using Euler's method, not in Specialist Maths. Also, homogeneous DE using substitution of y = vx ,and using integrating factor, are not in Specialist Maths.

		Examine slope fields of first-order differential equations.Reconstruct a graph from a slope field both manually and using graphics software					✓	
		Formulate differential equations in contexts where rates are involved. Use separable differential equation			✓		✓	
		Use the logistic differential equation			✓			All of AHL 5.19 Maclaurin series content not in Specialist Maths.
	Subtopic 6.3: Pairs of varying quantities — polynomials of degree 1 to 3	Consider examples of applications to: • uniform motion • vector interpretation					✓	
		objects in free flight			✓			
		For a moving point $(x(t),\ y(t))$, the vector of derivatives $\boldsymbol{v} = [x'(t),\ y'(t)]$ is naturally interpreted as its instantaneous velocity			✓			
		The Cartesian equation of the path of the moving point can be found by eliminating *t*						
		The velocity vector as tangent to the curve traced out by a moving point						
		• The speed of the moving point is the magnitude of the velocity vector, that is, $\sqrt{x'^2(t)+y'^2(t)} = \sqrt{\boldsymbol{v}\bullet\boldsymbol{v}}$			✓		✓	
		Find the arc length along parametric curves					✓	
		A point moving with unit speed around the unit circle can be described using the moving position vector $\boldsymbol{P}(t) = [\cos t,\ \sin t]$						
		Consider the speed of moving around other circles with other speeds						
		Other forms of parametric equations $\boldsymbol{P}(t) = [x(t), y(t)]$ where $x(t)$ and $y(t)$ are trigonometric functions that may not result in circular motion					✓	

								From the Analysis course:AHL 5.12, 5.13 in terms of continuity, differentiability, convergence, divergence and L'Hopitals rule not covered in Specialist Maths.
								From the Apps Course: 3.14, 3.15, 3.16 not covered in Specialist Maths. Also the component vector sections of 3.13. Calculus topics 5.17 is covered in part, but 5.18 is not covered in Specialist Maths.
√*	partially included							
√	included							